# Competitive Product Tests under Minimum Quality Standards

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**Abstract**

We consider a vertical oligopoly market in which (i) two firms have their products tested publicly before launch, and (ii) a minimum quality standard (MQS) is imposed. Firms choose the accuracy of their product tests, balancing two competing incentives: hiding information makes it easier to pass the MQS, while revealing information softens price competition through differentiation. In the unique symmetric equilibrium, each firm chooses a test that fully reveals high qualities and pools middle qualities around the MQS. For the regulator, the MQS reduces efficiency through reduced trade, though overall consumer surplus is increased due to intensified price competition.

*JEL Classification Numbers:* D43, D83, L13, L15

*Keywords:* Quality disclosure; competitive information design; vertical differentiation; minimum quality standard

# 1 Introduction

In many markets consumers have little information about the quality of a new product before or sometimes even after it is launched. To resolve the lemons problem and facilitate trade firms usually engage in various marketing strategies to disclose their quality information to the market. In the recent decades, the increasing availability of credible testing experts (e.g., IncoTest), third-party product reviewers (e.g., *Consumer Reports*) and widely adopted online reviews (e.g., Amazon reviews, consumerreview.com) enables the firms to disclose quality information in a public and credible, yet flexible way.

This paper studies the effect of competition and policy intervention on firms' choices of disclosure strategies in vertical oligopoly markets. In our framework, two competing firms can each choose to have a new product tested publicly before launch. We model the test as a publicly observable information structure that maps the quality of a product to a signal, or a score, without imposing additional restriction on it. Such a flexibility assumption and that the firms have full control over their own test can be rationalized by choosing from different third party tests available in the market with different coarseness and toughness, or even conducting their own pre-registered laboratory experiments. After the test results are publicized, the firms launch their products and are involved in a pricing game constrained by a minimum quality standard (MQS), in that a firm can obtain the licence to operate in the market and launch the product only if the score exceeds the MQS. When one or both firms pass the test and become available in the market, a unit mass of consumers with heterogeneous tastes for qualities decide whether and which product to purchase.

We find that in the unique symmetric equilibrium both firms adopt a simple monotone partitional strategy: the qualities within a neighborhood of the minimum standard are pooled, while all other qualities are perfectly revealed.[1] In other words, each firm chooses a test that conceal information when the realized quality is close to the MQS, so that the market participants cannot distinguish between two different qualities within this region. In particular, if the MQS is sufficiently low, all qualities at the bottom are pooled so that both firms always pass the test and the induced market structure is always duopoly. As the standard increases, one or both firms exit in

[1]The form of information revelation at the lower end is not unique. This is because a firm is excluded whenever the score falls strictly below the MQS, and thus the disclosure strategy in low states does not affect the market outcome. The symmetric equilibrium is essentially unique up to this payoff-irrelevant multiplicity.

case of a failure to meet the standard after the test. When the standard is sufficiently high, each firm pools all high states such that it either launches the product with a score right at the standard, or exits the market.

The equilibrium structure reflects two countervailing economic forces. Without an MQS, both firms have incentive to increase test precision since more information leads to higher level of perceived quality differentiation, and, hence, it mitigates price competition. With an MQS, however, a firm is excluded and earns nothing whenever the score is below the standard, but earns strictly positive profit if it passes (either as a monopolist or as a low-quality firm in the duopoly). The discreteness in profit creates a potential gain from local concealment: the firm can increase the chance of passing by pooling some low quality states, in which it would have failed under full information, with higher states.

We further investigate the welfare implication of the MQS policy. It is intuitive that imposing a non-trivial MQS hurts both firms, since it intensifies the price competition through partial concealment and sometimes induces exclusion from the market. In fact it is also socially inefficient if a regulator/planner assigns equal Pareto weights to consumer surplus and producer surplus, because the prices are pure transfers and do not affect the social welfare. In this case the only goal of the planner is the matching efficiency, that is, consumers who are more (less) sensitive to quality purchase from the firm with higher (lower) quality. Without an MQS the firms reveal full information in equilibrium and the planner can achieve the upper bound of social efficiency through a sorting pricing equilibrium. When the planner cares enough about consumer surplus, the impact of competition intensification becomes a major concern. If the MQS is sufficiently close to the lower bound of the quality, the regulator wants to increase it in order to generate more equilibrium pooling and, therefore, more competition. But if the MQS becomes too high then the possibility of a monopoly or of market shut-down becomes significant. Thus, the optimal MQS is interior.

Our results shed light on why firms might sometimes practice cutoff disclosure strategies in which only high qualities are fully revealed, even when testing is costless and the information strategy is not restricted to “all or nothing” as in disclosure games. We also highlight the interaction between a minimum quality standard and the firms’ marketing strategy. Introducing a minimum standard could be socially beneficial even without considering its effect on firms’ investment in quality. On the technical side, by modelling the marketing strategy as a choice of information structures, we bridge the recent development in information design with the classical topic of quality disclosure in the IO literature. In particular, we are among the first studies (see also Boleslavsky et al., 2019; Yang, 2020) that consider competitive persuasion with post-persuasion market interactions. In a model with independent and continuous state space, we demonstrate the construction of a competitive persuasion equilibrium within the class of games with piece-wise linear (*ex post*) payoffs.

Section 2 discusses how our analysis relates and contributes to the existing literature. Section 3 outlines the model setup. Section 4 provides the main result on the market equilibrium. Section 5 presents results on welfare effects of the policy intervention. Section 6 discusses and concludes. All proofs for secondary results can be found in the appendix.

## 2 Related Literature

This paper is related to several strands of literature. First, it contributes to the literature on disclosure games in oligopoly markets. Board (2009) investigates competitive quality disclosure in an uncovered vertical duopoly with heterogeneous consumers. Levin et al. (2009), on the other hand, consider quality disclosure with both horizontal and vertical differentiation, but in absence of consumer heterogeneity. Our model, like Board (2009), is based on the vertical differentiation models developed by Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). In contrast, we focus on the case of covered market and consider an alternative communicating channel, Bayesian persuasion, motivated by the extensive use of public tests or certifications in the market.

Canidio and Gall (2019) considers a model with covered market similar to ours. In their model each firm can generate a public signal correlated to its quality at a fixed cost before pricing. Their information structure departs from the paradigm of voluntary disclosure, but is still exogenously fixed, whereas in our model information structures are completely endogenous. In addition, we assume costless public tests, while the firms face a policy constraint, the minimum quality standard.

This paper belongs to a growing literature on Bayesian persuasion and information design (Kamenica and Gentzkow, 2011; Gentzkow and Kamenica, 2016a; Kolotilin et al., 2017; Au and Kawai, 2019,0; Koessler et al., 2018; Garcia, 2018; Dworczak and Martini, 2019; Arieli et al., 2019; Kleiner et al., 2020) and their applications in market environments (Gill and Sgroi, 2012; Roesler and

Szentes, 2017; Boleslavsky et al., 2017; Armstrong and Zhou, 2019; DeMarzo et al., 2019; Zapechelnyuk, 2020). In particular we build a model of competitive persuasion with independent and continuous state space, which is followed by a subsequent pricing game. A closely related paper is Boleslavsky et al. (2019), which studies a similar model but in an horizontal oligopoly in which consumers face uncertainty in independent match values rather than common product qualities. Moreover, the pricing and information strategies are chosen simultaneously in their model while we consider price competition after the revelation of public signals.[2] In a companion paper (Yang, 2020) we study the same baseline model of vertical differentiation, but focus on an uncovered market without policy interventions. We show that the demand effect becomes a main concern in the uncovered case, which is absent in the current paper.

Finally, the role of minimum quality standards in vertically differentiated markets is considered in a small but long-standing literature (Leland, 1979; Ronnen, 1991; Crampes and Hollander, 1995; Lutz et al., 2000; Buehler and Schuett, 2014). A common insight due to Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) is that firms can relax price competition by enlarging product differentiation. Thus, adopting an MQS can be socially beneficial because it limits the range of quality differentiation and facilitates higher quality provision.[3] For instance, Ronnen (1991) demonstrates these effects of MQS in a model with fixed quality investments. Crampes and Hollander (1995) address the same question when quality-provision costs are variable and sufficiently convex. In contrast our paper focuses on the interaction between the MQS and the firms' disclosure strategy, while taking the true quality distributions and realizations as exogenous.

## 3 Model

Two firms launch new products aimed at a unit mass of risk neutral consumers. The consumers are heterogeneous in their sensitivity to quality, represented by $\theta$. We assume $\theta$ is uniformly distributed with support $[\underline{\theta}, \bar{\theta}]$. Each consumer has unit demand and receive utility $q_i\theta - p_i$ when purchasing the product $i$ with $q_i$ at price $p_i$, and zero otherwise.

[2]The sequential structure is more plausible and interesting in the stylized vertical oligopoly model. If instead the firms choose prices before the signals are realized, the symmetric equilibrium has to feature zero prices for both firms that resembles the classical Bertrand equilibrium.

[3]A binding MQS forces the low quality firm to increase its quality, which in turn drives the high quality firm to increase quality due to the strategic complementarity between quality choices of different firms.

Firm $i \in \{1,2\}$ produces a new product with quality $q_i \in [\underline{q}, \bar{q}]$ at zero marginal cost. The qualities are independently distributed with identical cumulative distribution function $F$ that is uniform. $F$ is common knowledge, while neither the firms nor the consumers know the realization of $(q_i, q_j)$ *a priori*.

There are three stages. Before releasing, each firm subjects its product to a pre-launch public test. A test $(S_i, \tau_i)$ is represented by an information structure that consists of a signal (score) space $S$ and a mapping $\tau_i : q_i \mapsto \Delta(s_i)$ from the quality to a distribution of scores. After the a score $s_i$ is generated, the society forms a posterior belief $\mu_{s_i}$ about $q_i$ using Bayes rule. Without loss of generality we assume the score takes a literal meaning, such that $s_i = E_{\mu_{s_i}}(q_i)$.[4] We assume no particular structure of the test and each firm can design its own testing strategy in a flexible way. Both the tests $\{(S_i, \tau_i)\}_{i=1,2}$ and the realized scores $\{s_i\}_{i=1,2}$ are publicly observable.

After the test stage, the firms launch their goods facing an exogenously imposed minimum quality standard (MQS) $s_0$. Firm $i$ can launch its good only if $s_i \geq s_0$. If neither firms are qualified, the game ends. If exactly one firm is qualified, it operates as a monopolist. If both firms are qualified, the game proceeds to the price competition stage in which two firms simultaneously set prices $p_i$ and $p_j$. In the last stage, the consumers make purchase decisions based on the price(s) and test results.

In this paper we focus on the covered but non-preempted market in the duopoly case, that is, all consumers purchase and both firms face non-zero demand.[5] Thus we impose the following two assumptions:

**Assumption 1** (Covered market)**.** $\dfrac{\bar{q} - \underline{q}}{\underline{q}} \leq \dfrac{3\underline{\theta}}{\bar{\theta} - 2\underline{\theta}}$.

**Assumption 2** (Sufficient taste heterogeneity)**.** $\bar{\theta} \geq 2\underline{\theta}$.

These two assumptions are in line with those made in Tirole (1988). Assumption 1 ensures that for any realized scores (even in the most extremely differentiated case) all consumers buy one of the

[4] This is without loss of generality because the consumers only care about the expected quality given a certain signal, that is, the posterior mean. Thus, we can combine any two signals that induce the same posterior mean (not necessarily the same posterior) and relabel the signal as $s_i = E_{\mu_{s_i}}(q_i)$.

[5] In the companion paper Yang (2020) we investigate the competitive test design when the duopoly market is not covered. For instance, the duopoly market is never covered when $\underline{\theta} = 0$ since the consumer with type $\underline{\theta}$ does not purchase any product unless the price is 0. In this case the market coverage is endogenously determined by the perceived qualities and, hence, the information strategy, resulting an additional demand effect of quality disclosure.

two brands. Assumption 2 says that the consumer taste for quality is sufficiently heterogeneous. It guarantees that in equilibrium the low score firm also serves a positive fraction of the market. The role of these two assumptions becomes clear once we derive the pricing equilibrium.

The solution concept in this paper is strong Perfect Bayesian Equilibrium. A strategy profile $(\tau_i^*, p_i^*(s))_{i=1,2}$ constitutes an sPBE if 1) $(p_1^*(s), p_2^*(s))$ forms a pricing equilibrium given any pair of realized $s = (s_1, s_2)$, 2) $\tau_i^*$ is firm $i$'s optimal information strategy given $\tau_{-i} = \tau_{-i}^*$ and $(p_1, p_2) = (p_1^*, p_2^*)$ for any sub-game following realization $(s_1, s_2)$, and 3) the public posterior belief following any $s$ is derived through Bayes rule whenever possible. We focus on the symmetric equilibrium.

# 4 Main Results

## 4.1 Pricing equilibrium

We first examine the pricing equilibrium for a fixed pair of realized scores $(s_1, s_2)$. If only one firm passes the test and remains in the market, the consumers purchase if and only if $E(q\theta - p|s) \geq 0$. The linear structure reduces the condition to $s\theta - p \geq 0$. The monopolist faces demand $D^m = Pr(\theta \geq \frac{p}{s}) = \frac{1}{\bar{\theta}-\underline{\theta}}(\bar{\theta} - \frac{p}{s})$. The monopoly price is given by the first order condition:

$$p^m = \frac{\bar{\theta}}{2}s.$$

Thus the monopoly profit is $\pi^m(s) = p^m D^m(p^m) = \dfrac{\bar{\theta}^2}{4(\bar{\theta} - \underline{\theta})}s$.

The duopoly case follows the textbook treatment in Tirole (1988). We label the two firms $h$ and $l$ according to the score ranking. Given revealed scores $(s_h, s_l)$ and prices $(p_h, p_l)$ the consumers sort into two subsets $[\underline{\theta}, X]$ and $[X, \bar{\theta}]$ such that high type consumers $(\theta > X)$ purchase from the high score firm, low type consumers $(\theta < X)$ purchase from the low score firm, and the threshold type $(\theta = X)$ is indifferent:

$$s_h X - p_h = s_l X - p_l \Rightarrow X = \frac{p_h - p_l}{s_h - s_l}.$$

The firms face demand $D^h = \frac{1}{\bar{\theta}-\underline{\theta}}(\bar{\theta} - X)$ and $D^l = \frac{1}{\bar{\theta}-\underline{\theta}}(X - \underline{\theta})$. The pricing equilibrium is easily solved through first order conditions:

$$p_h^d = \frac{(2\bar{\theta} - \underline{\theta})}{3}(s_h - s_l),\ p_l^d = \frac{(\bar{\theta} - 2\underline{\theta})}{3}(s_h - s_l).$$

Accordingly, the equilibrium cutoff is $X^d = \frac{\bar{\theta}+\underline{\theta}}{3}$ and the equilibrium profits are given by

$$\pi_h^d(s_h, s_l) = \frac{(2\bar{\theta} - \underline{\theta})^2}{9(\bar{\theta} - \underline{\theta})}(s_h - s_l),\ \pi_l^d(s_h, s_l) = \frac{(\bar{\theta} - 2\underline{\theta})^2}{9(\bar{\theta} - \underline{\theta})}(s_h - s_l).$$

Note that in the covered market case the equilibrium segmentation determined by $X$ does not depend on the perceived qualities $(s_1, s_2)$. The demand for each firm depends only on the ordinal ranking of qualities, so both the equilibrium prices and the equilibrium profits are linear functions of the perceived quality differentiation $(s_h - s_l)$.[6] When $(s_h - s_l)$ increases, the firms are more differentiated and hence less competitive, leading to higher prices and profits for both firms. This fact is convenient since it implies that the information strategies in the first stage only affect the identity of high and low firms and their equilibrium prices, but not demand.

## 4.2 Information equilibrium

Note the consumers' choices depend only on the conditional expected qualities given $(s_1, s_2)$ (hence the scores themselves) but not on any other distributional properties of the induced posterior beliefs. In consequence, the firms' profits also depend only on the posterior means. Following the literature on Bayesian persuasion and information design (e.g., Gentzkow and Kamenica, 2016b; Kolotilin, 2018), each firm's choice $(S_i, \tau_i)$ is equivalent to the choice of a distribution $G_i(s_i)$ over the posterior means $s_i$ subject to the constraint that $G_i$ is a mean-preserving contraction of $F$,[7] denoted as $G_i \in \text{MPC}(F)$.

$$\max_{G_i \in MPC(F)} \int_{\underline{q}}^{\bar{q}} \int_{\underline{q}}^{\bar{q}} \pi_i(s_i, s_{-i}) dG_{-i}(s_{-i}) dG_i(s_i)$$

[6]This is because the best responses of both $h$ and $l$ firms are proportional to $(s_h - s_l)$. See Canidio and Gall (2019) for a detailed discussion of the (price) best responses under general distributional assumptions.

[7]Equivalently, $F$ is a mean-preserving spread of $G_i$, defined as follows: $\int_0^t F_i(x)dx \geq \int_0^t G_i(x)dx$ for any $t \in [0, 1]$ and $\int_0^1 F_i(x)dx = \int_0^1 G_i(x)dx$.

where the *ex post* profit is given by

$$\pi_i(s_i, s_{-i}) = \mathbb{1}_{\{s_i \geq s_0 > s_{-i}\}} \pi^m(s_i) + \mathbb{1}_{\{s_{-i} > s_i \geq s_0\}} \pi_l^d(s_i, s_{-i}) + \mathbb{1}_{\{s_i \geq s_i \geq s_0\}} \pi_h^d(s_i, s_{-i}).$$

Each firm plays one of four possible roles depending on $(s_1, s_2)$: 1) the excluded firm when $s_i < s_0$, 2) the monopoly firm when $s_{-i} < s_0 \leq s_i$, 3) the low quality firm in the duopoly when $s_0 \leq s_i \leq s_{-i}$, and 4) the high quality firm in the duopoly when $s_0 \leq s_{-i} \leq s_i$. Note in the knife-edge case between case 3) and 4) the firms are facing the extreme Bertrand competition and equilibrium prices are driven to zero. Firm $i$ chooses $G_i$ to determine the distribution $G = G_1 \times G_2$ over $(s_1, s_2)$ and maximize its expected profit $E_G(\pi_i(s_1, s_2))$ given the choice of the opponent firm. Figure 1 demonstrates the shape of $\pi_i(s_i, s_{-i})$ as a function of $s_i$ for a fixed $s_{-i}$. Depending on whether firm $-i$ passes the test, firm $i$'s *ex post* profit follows one of the two piece-wise linear structures.

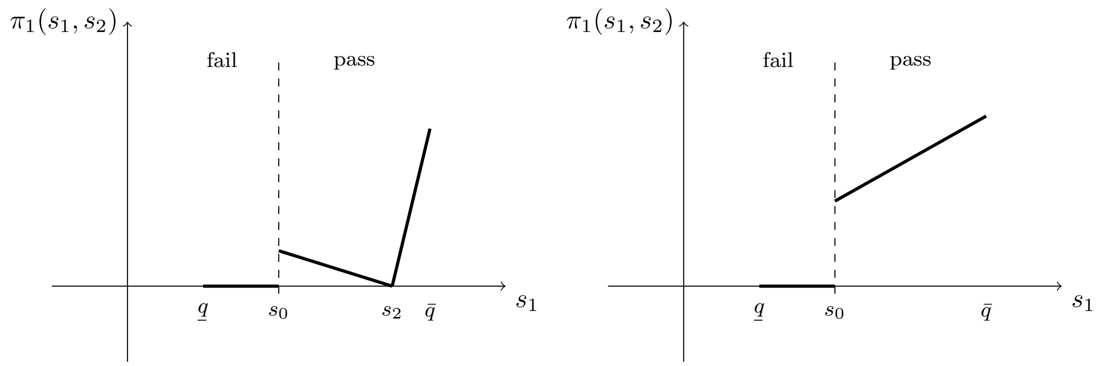


Figure 1: *Ex post* profit of firm 1 given $s_2$ (a) $s_2 \geq s_0$ (b) $s_2 < s_0$

We show that in equilibrium both firms follow an information strategy in the form of monotone partitions specified in the definition below.

**Definition 1.** *An information strategy is within the class of $G^{\delta}$ if there exists a $\delta$ such that*

$$G^{\delta}(s)=\begin{cases}F(s) & \text{if } s\leq s_0-\delta\\ F(s_0-\delta) & \text{if } s_0-\delta<s<s_0\\ F(s_0+\delta) & \text{if } s_0\leq s<s_0+\delta\\ F(s) & \text{if } s\geq s_0+\delta\end{cases}$$

*in which either $[\underline{q},s_0-\delta)$ or $(s_0+\delta,\bar{q}]$ or neither is empty.*

**Theorem 1.** *For any $s_0\in[\underline{q},\bar{q}]$ there exists a symmetric information equilibrium $(G_1^*,G_2^*)$ such that $G_1^*=G_2^*=G^{\delta^*}$ in which $\delta^*\in(0,\min\{s_0-\underline{q},\bar{q}-s_0\}]$ is uniquely determined. In particular, there exists a unique pair of $(s_0^l,s_0^u)$ such that $s_0^l<\dfrac{\bar{q}+\underline{q}}{2}<s_0^u$ and*

- $\delta^*=s_0-\underline{q}$ *when* $s_0\leq s_0^l$,
- $\delta^*<\min\{s_0-\underline{q},\bar{q}-s_0\}$ *when* $s_0^l<s_0\leq s_0^u$,
- $\delta^*=\bar{q}-s_0$ *when* $s_0>s_0^u$.

*Moreover this equilibrium is unique up to a payoff-irrelevant change in $G^{\delta^*}$ on $[\underline{q},s_0-\delta^*]$.*

**Remark.** In fact any $G$ such that $G|_{[\underline{q},s_0-\delta^*]}\in \mathrm{MPC}(F|_{[\underline{q},s_0-\delta^*]})$ and $G|_{[s_0-\delta^*,\bar{q}]}=G^{\delta^*}|_{[s_0-\delta^*,\bar{q}]}$ can arise in equilibrium. This is because each firm earns 0 in the interval $[\underline{q},s_0-\delta^*]$ and any change in the disclosure rule in this region does not affect $\Pi_i$ for either $i$.

**Corollary 1.** *The following strategies and beliefs constitute a symmetric sPBE: the equilibrium test structure is such that $S_i=[\underline{q},\bar{q}]$ and*

$$\tau(q_i)=\begin{cases}s_0 & \text{if } s_0-\delta\leq s\leq s_0+\delta\\ q_i & \text{otherwise}\end{cases},$$

*the pricing equilibrium is as specified in section 3.1, and the public posterior belief is consistent with Bayes rule on the equilibrium path (when $s_i\in[\underline{q},s_0-\delta^*]\cup\{s_0\}\cup[s_0+\delta^*,\bar{q}]$) and passive off the equilibrium path, that is, $\mu_{s_i}$ is a Dirac distribution at $\underline{q}$ for all $s_i\in(s_0-\delta^*,s_0)\cup(s_0,s_0+\delta^*)$.*

**Corollary 2.** *When there is no binding MQS, $s_0 = \underline{q}$, the unique sPBE features 1) full information in the first stage (so no off-equilibrium-path beliefs), and 2) pricing equilibrium as described in section 4.1.*

Theorem 1 provides an equilibrium characterization in terms of the distribution of scores $G$. Corollary 1 describes a test structure that implements the equilibrium $G^{\delta^*}$. The equilibrium test design has a monotone-partition structure and reduces to full revelation when the MQS does not exist (corollary 2). The intuition follows the literature on vertical oligopolies since Gabszewicz and Thisse (1979) and Shaked and Sutton (1982). The firms always benefit from less competition by enlarging the quality differentiation. In the current environment of exogenous quality but endogenous information, the firms can only choose a distribution over the possible scores subject to a mean-preserving contraction condition. Fix any strategy by the opponent $-i$, full information is a best response of firm $i$ when there is no MQS because revelation gives the highest possible level of expected differentiation. Thus the unique information equilibrium features full revelation and it is in fact a dominant strategy equilibrium.

When there is a non-trivial MQS, a new trade-off is introduced. On one hand, the firms still have the incentive to soften competition through enlarging differentiation in the duopoly case. On the other hand, each firm has an incentive to increase the probability of meeting the MQS. This is because the firm is excluded from the market and gets 0 profit following a failure ($s_i < s_0$), while meeting the MQS, even at the minimum level $s_i = s_0$, leads to a strictly positive profit. Such a profit discontinuity at $s_0$ creates an incentive for local concealment.

## 4.3 Proof sketch of Theorem 1

This subsection presents the proof sketch for our main theorem (all detailed proofs can be found in the appendix).

Throughout we make use of two results in the recent literature on linear persuasion problems by Dworczak and Martini (2019), Arieli et al. (2019) and Kleiner et al. (2020). These two results provide solutions to the single-sender problems and lay the foundation for our best response analysis.

Consider a linear persuasion problem $(F, \pi)$ with prior $F$ and payoff function $\pi$:

$$\max_{G \in \text{MPC}(F)} \int \pi(x) dG(x).$$

We summarize the optimal solutions in the following lemmas.

**Lemma 2.** *(Dworczak and Martini, 2019) If there exists a cdf $G$ and a convex function $\phi : [0,1] \to$ $\mathbf{R}$, such that*

*(1)* $\phi(x) \geq \pi(x)\ \forall\ x \in [0,1]$,

*(2) supp*$(G) \subseteq \{x \in [0,1] : \pi(x) = \phi(x)\}$,

*(3)* $\int_0^1 \phi(x) dG(x) = \int_0^1 \phi(x) dF(x)$*, and*

*(4) $G$ is a mean-preserving contraction of $G$.*

*Then $G$ is a solution to the optimal information design problem.*

**Definition 2** (Bi-pooling policy)**.** *A distribution $G \in MPC(F)$ is a bi-pooling distribution w.r.t. $F$ if there exists a collection of pairwise disjoint intervals $\{(a_i, b_i)\}_i$ such that*

1. *For each $i$, $G((a_i, b_i)) = F((a_i, b_i))$ and $|supp[G|_{(a_i,b_i)}]| \leq 2$.*
2. *For all $x \notin \cup_i (a_i, b_i)$, $G(x) = F(x)$.*

**Lemma 3.** *(Arieli et al., 2019; Kleiner et al., 2020) Every persuasion problem $(F, \pi)$ has an optimal bi-pooling policy.*

Lemma 2 provides a tractable graphic method to verify the optimality of a given information strategy $G$. Lemma 3 characterizes the set of extreme points in the linear persuasion problems. We first check the best response conditions (lemma 4) and verify the proposed equilibrium characterization (lemma 5). The existence directly follows the construction.

**Lemma 4** (best response)**.** *Assume firm 2 follows the proposed strategy with arbitrary $\delta \leq \min\{\bar{q} - s_0, s_0 - \underline{q}\}$, there exists a $\delta'$ such that it's a best response for firm 1 to choose $G_1 = G^{\delta'}$.*

**Lemma 5** (fixed point)**.** *For any $s_0$ there exists a unique $\delta^*$ such that the proposed strategy profile $(G^{\delta^*}, G^{\delta^*})$ constitutes a symmetric equilibrium. In particular, there exists a unique $s_0^l \in (\underline{q}, \frac{\underline{q}+\bar{q}}{2})$ and a unique $s_0^u \in (\frac{\underline{q}+\bar{q}}{2}, \bar{q})$, such that the equilibrium features*

1. $\delta^* = s_0 - \underline{q}$ *when* $s_0 \leq s_0^l$,

2. $\delta^* \in \big(0, \max\{s_0 - \underline{q}, \bar{q} - s_0\}\big)$ *when* $s_0^l < s_0 < s_0^u$, *and*

3. $\delta^* = \bar{q} - s_0$ *when* $s_0 \geq s_0^u$.

To demonstrate the structure of the best responses, we first characterize firm 1's interim payoff function $\Pi_1(s_1) = \int \pi_1 dG_2$ when firm 2 follows the proposed strategy $G_2 = G^\delta$. Figure 2 illustrates the shape of $\Pi_1$: it is 0 when $s_1 < s_0$, linear and increasing when $s_1 \in [s_0, s_0 + \delta)$, and convex and increasing when $s_1 \in [s_0 + \delta, \bar{q}]$. To verify firm 1's best response, we simply apply lemma 2 and let $\phi$ be the maximum of $\Pi_1$ and the dashed blue curve. For instance, in the left panel firm 1's best response is $G_1 = G^{\delta'}$ with $\delta' = s_0 - s'$:

- $\phi$ is convex and $\phi \geq \Pi_1$,
- supp$G_1 = [\underline{q}, s'] \cup \{s_0\} \cup [2s_0 - s', \bar{q}] \subset \{s_1 : \phi(s_1) = \Pi_1(s_1)\}$,
- $E_F(\phi(s_1)) = E_{G^{\delta'}}(\phi(s_1))$ because $G^{\delta'} = F$ for all $s_1 \notin (s_0 - \delta', s_0 + \delta')$ and

$$E_F(\phi(s_1))|_{(s_0-\delta', s_0+\delta')} = F((s_0 - \delta', s_0 + \delta'))\phi(s_0) = E_G(\phi(s_1))|_{(s_0-\delta', s_0+\delta')}.$$

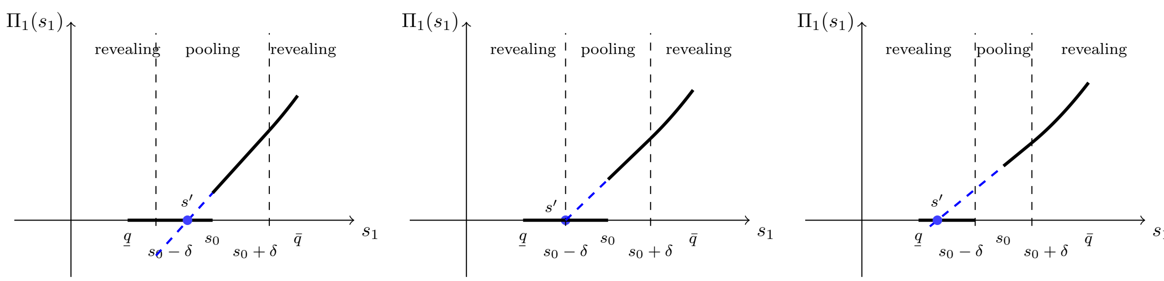


Figure 2: Interim profit of firm 1 with (a) $s' > s_0 - \delta$, (b) $s' = s_0 - \delta$ and (c) $s' < s_0 - \delta$

We can verify in a similar way that $\delta' = s_0 - s' = \delta$ in the central panel and $\delta' \in (0, s_0 - s')$ in the right panel. Observe that when $\delta$ is large (small), firm 1 response by a smaller (larger) $\delta'$, implying that the best response correspondence is a contraction mapping. Thus there exists a unique $\delta$ (the central panel) that constitutes the symmetric equilibrium.

The arguments above cover the cases when $s_0$ is at an intermediate level. When $s_0$ is close to $\underline{q}$ or $\bar{q}$, the choice of fixed point $\delta$ is capped by $s_0 - \underline{q}$ or $\bar{q} - s_0$. In those cases we can simply set $\delta^*$ to equal the bounds and rebuild the equilibrium. The structure is the same as the intermediate $s_0$ case except that either $[\underline{q}, s_0 - \delta]$ or $[s_0 + \delta, \bar{q}]$ is empty. Figure 3 demonstrates these two corner equilibria.

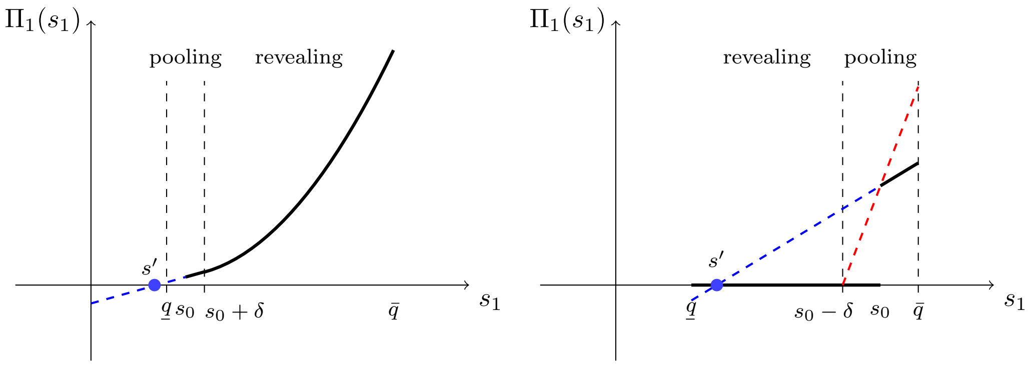


Figure 3: Interim profit of firm 1 in corner equilibrium (a) low $s_0$, (b) high $s_0$

Combining lemma 4 and lemma 5 we can prove the equilibrium characterization in Theorem 1. The uniqueness is established step by step as follows. According to lemma 3, the best response of firm 1 to any information strategy $G_2$ takes the bi-pooling structure unless $\Pi_1$ is locally linear, wherein firm 1 is indifferent among any $G \in \text{MPC}(F)$ and the best response can be arbitrary. We rule out all other possibilities in lemma 6, 7 and 8.

First of all, we show that full information does not constitute an equilibrium (lemma 6). The proof is straightforward: when firm 2 is fully revealing, firm 1 faces an interim expected profit function that is 0 when $s_1 < s_0$ and strictly positive when $s_1 = s_0$. Hence, full information is not a best response. Firm 1 has an incentive to deviate to a strategy that involves at least partial concealment in the neighborhood of $s_0$.

**Lemma 6** (no full revelation equilibrium)**.** *$G_i = G_j = F$ is not an equilibrium for any $s_0 > \underline{q}$.*

Next we show that there exists no symmetric equilibrium such that $G$ features an atom above $s_0$. That is, local pooling arise only at $s_0$ in a symmetric equilibrium. To prove this result, let firm 2 pool a neighborhood of an arbitrary state $s > s_0$. We can show that firm 1's interim profit

$\Pi_1(s_1)$ within this neighborhood $(s-\epsilon, s+\epsilon)$ is convex, so according to lemma 2 the best response cannot involve local pooling around $s$. This rules out all symmetric equilibria in local uni-pooling or bi-pooling strategies other than $G^\delta$. Following the same logic, we can rule out any $G$ with a mass point at any $s' > s_0$ (corollary 3).

**Lemma 7** (no pooling besides $s_0$). *For any candidate symmetric equilibrium, $G$ does not involve local uni-pooling at any $s' > s_0$ or any bi-pooling sub-intervals.*

**Corollary 3** (no mass point besides $s_0$). *For any candidate symmetric equilibrium, $G$ does not involve a mass point at any $s' > s_0$.*

Lemma 6 and 7 shows that within the class of bi-pooling strategies, the only possible symmetric equilibrium is $G^{\delta^*}$ proposed in Theorem 1. There is one more case to rule out, which we refer to as the "matching-pennies" type equilibrium. Note that it is without loss to restrict attention to bi-pooling policies when we consider a single-sender problem, because bi-pooling policies cover all extreme points of linear persuasion problems. This is not without loss when we consider the equilibrium in a game. As demonstrated by Boleslavsky et al. (2019) and Yang (2020), a strategy $G \in MPC(F)$ constitutes a symmetric equilibrium when it can make $\Pi_1(s_1) = \int \pi_1(s_1, s_2) dG(s_2)$ linear over a subset of $(\underline{q}, \bar{q})$ even if it is not a bi-pooling strategy. This is because when $\Pi_i$ is linear firm $i$ is indifferent among various information strategies, which justifies the optimality of the strategy $G_{-i} = G$ that leads $\Pi_i$ to be linear. Such an equilibrium resembles the intuition of the mixed strategy equilibrium in matching-pennies games.

The next lemma rules out such "mixed" strategy equilibria.[8] The proof idea follows the previous lemmas. Consider any strategy $G_2$ that is continuous but different from $F$ on a sub-interval of $[s_0, \bar{q}]$. Firm 1 best responds by choosing the same distribution only if $\Pi_1(s_1)$ is linear in $s_1$ in the same sub-interval. Such a possibility is falsified by the piece-wise linearity of $\pi_1$.

**Lemma 8** (no "matching-pennies" equilibrium). *For any candidate symmetric equilibrium and any interval $[s_l, s_u]$, there exists a finite partition of this interval such that either $G = F$ or $G$ takes a constant value. No other strategies can be supported in any symmetric equilibrium.*

[8] It is technically different from the mixed strategy equilibrium defined in a finite game since the strategy space is a subset of all possible distributions MPC($F$). The underlying idea that each player chooses her strategy to make the opponent indifferent, however, is the same.

# 5 Welfare analysis

Given the equilibrium characterization we can consider the welfare impact of the minimum quality standard $s_0$. Assume the regulator chooses an MQS to maximize a weighted sum of the producer surplus (PS) and the consumer surplus (CS):

$$\max_{s_0} \alpha E_{G^*}[CS(s_h, s_l)] + (1-\alpha) E_{G^*}[\pi_h(s_h, s_l) + \pi_l(s_h, s_l)] \ .$$

where $CS(s_h, s_l) = \mathbb{1}_{\{s_l < s_0 \le s_h\}} CS^m(s_h) + \mathbb{1}_{\{s_0 \le s_l \le s_h\}} CS^d(s_h, s_l)$, $\pi_h(s_h, s_l)$ and $\pi_l(s_h, s_l)$ are given in section 4.1, and $CS^m$ and $CS^d$ are computed as follows:

$$\begin{aligned} CS^m(s) &= \int_{\bar\theta/2}^{\bar\theta} [\theta s - p^m(s)] \, dH(\theta), \\ CS^d(s_h, s_l) &= \int_{\underline\theta}^{X} \left[\theta s_l - p^d(s_h, s_l)\right] dH(\theta) + \int_{X}^{\bar\theta} \left[\theta s_h - p^d(s_h, s_l)\right] dH(\theta). \end{aligned}$$

Note that we can directly write the surplus terms as functions of $(s_1, s_2)$ since the *ex post* profits are already shown to be independent of true qualities and the *ex post* consumer surplus $CS(q_1, q_2)$ is linear in the true qualities.[9] Figure 4 illustrates the score distribution and the associated market structure when $\delta^*$ is interior.[10] For instance all realizations in $(s_0 - \delta^*, s_0 + \delta^*)^2$ are pooled at $(s_0, s_0)$, resulting in a duopoly market with extreme Bertrand competition. The *ex ante* surplus terms can be computed according to this distribution.

We consider two special cases: a social-welfare-maximizing regulator with equal Pareto weights $\alpha = \frac{1}{2}$ and a consumer-surplus-maximizing regulator with $\alpha = 1$. Proposition 1 shows that in the first case any non-trivial MQS must be socially inefficient.

**Proposition 1.** *When $\alpha = \frac{1}{2}$, the regulator optimally chooses $s_0 = \underline{q}$.*

The underlying intuition is straightforward. When $\alpha = \frac{1}{2}$, the prices are just transfers within the society and have no impact on the social welfare, regardless of the market structure. The regulator

[9] For instance, in a monopoly market with true quality $q$ and score $s$ we have $CS^m(q, s) = \int_{\bar\theta/2}^{\bar\theta} [\theta q - p^m(s)] \, dH(\theta)$. Following any score realization that leads to such a monopoly market, the consumer surplus $CS^m(s) = E\left[CS^m(q, s) | \, s\right] = \int_{\bar\theta/2}^{\bar\theta} [\theta s - p^m(s)] \, dH(\theta)$. The same logic holds in the duopoly case.

[10] Here we assume the firms are pooling instead of revealing all states below $s_0 - \delta^*$, but as discussed in section 4.2 this is payoff irrelevant.

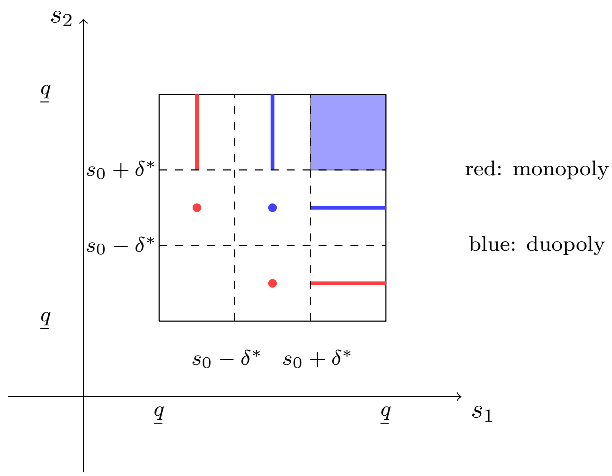


Figure 4: Equilibrium score distribution and market structure with interior $\delta^*$.

only cares about value $\theta q$ generated from the trade. Since the quality and the consumer type present complementarity, the regulator prefers sorting outcomes, that is, high (low) type consumers buy high (low) quality products, to randomizing for fixed market shares.[11] When $s_0 \leq s_0^l$, the market is always duopoly. Any $s_0 > \underline{q}$ leads to pooling in the region $(s_0 - \delta^*, s_0 + \delta^*)^2$. In this situation the two products have the same perceived quality and all consumers randomize over two firms, leading to both mismatch (high type consumers might end up with a low quality product and vice versa) and smaller market share for the high quality product (in full information equilibrium the high quality firm serves more than one half of the consumers since $X = \frac{\bar{\theta}+\underline{\theta}}{3} < \frac{\bar{\theta}+\underline{\theta}}{2}$). When $s_0$ is even larger so that the information equilibrium becomes interior, the mismatch effect grows and there is an additional source of inefficiency, possibility of no trade, either when both firms fail the test or only one monopolist serves the market. Given these concerns the welfare maximizing regulator prefers no MQS so that full information equilibrium results.

**Proposition 2.** *When $\alpha = 1$, the regulator optimally chooses a non-trivial MQS $s_0 \geq s_0^l$.*

To understand the intuition, note that the consumers enjoy the gain from trade and incurs the loss from payments. On the one hand, a non-trivial $s_0$ reduces consumer surplus in the same way as it reduces social welfare. On the other hand, it leads to partial concealment and restricts the

[11]Of course the First Best is that all consumers purchase from the high quality firm. However this is not feasible since the regulator has no control over the trading process.

scope of differentiation. When both firms are pooled at $s_0$, the consumers pay zero due to the extreme Bertrand competition. Proposition 2 shows that when no exclusion is involved (the market structure is always duopoly), the gain from intensified price competition always dominates the loss from inefficient matching, so the consumer surplus increases with the MQS. When $s_0$ exceeds $s_0^l$, possibility of market shut-down or monopoly leads to higher allocation inefficiency and, hence, might harm the consumers in net.

The two propositions directly imply that the regulator introduces a non-trivial MQS only if she weighs consumer surplus sufficiently high.

**Corollary 4.** *The regulator optimally chooses a non-trivial minimum quality standard $s_0 > \underline{q}$ only if $\alpha$ is sufficiently close to 1.*

# 6 Discussion

**General distributional assumption.** Although we assume that both the quality distribution $F$ and the consumer taste distribution $H$ are uniform, the main result directly carries over if we consider general distributions. This is because the qualitative results depend crucially on the piece-wise linearity of the *ex post* equilibrium profits, which relies on neither $F$ nor $H$ being uniform.

In particular, relaxing $H$ results in no closed form of the cutoff consumer type $X$. However, as long as the density $h$ is log-concave and satisfies the generalized covered duopoly market assumptions, equilibrium $X$ can be uniquely characterized and more importantly, it only depends on $H$.[12] As a result, equilibrium prices are still linear in the perceived differentiation, leading to no qualitative change in the basic structure of the information game. As for the distribution of quality $F$, note that no proof of the main theorem relies on the uniform distribution assumption of $F$. We assume $F$ to be uniform only for expositional simplicity, especially in the welfare analysis.

**Uncovered market.** The covered market assumption is crucial since it results in the constant market share property and hence the linearity of *ex post* equilibrium profits. Extension to uncovered market brings new trade-offs as well as technical challenges. In a companion paper (Yang, 2020) we study the uncovered case without policy interventions. We find that full revelation is no longer a

[12] See Canidio and Gall (2019) for a complete characterization.

dominant strategy even without a minimum quality standard. For example a firm would optimally choose a *pass or fail* strategy if the opponent fully conceals the quality. This is because the market shares are no longer fixed in the uncovered market and the equilibrium payoff becomes piece-wise concave instead of piece-wise linear. The competition, however, drives both firms to disclose more and full revelation arise in equilibrium in some but not all circumstances.

**Who controls the information?** Our model assumes that the firms have full control over and no cost in their own information structure. The regulator plays a relatively passive role, in the sense that she can only affect the market through the choice of the minimum quality standard.

Alternatively, firms might face mandatory disclosure policies that are directly regulated in some markets. For instance, food manufacturers must conform to FDA requirements regarding nutritional information disclosure. In such a situation we can consider a regulator directly designs a public test structure that is by nature two-dimensional. In our model setup, the resulting optimal information structure is simple:

**Proposition 3** (Regulator-optimal test)**.** *If the regulator has full control over the test structure,*

1. *full revelation is optimal if the regulator is welfare maximizing with equal weights, and*
2. *full concealment is optimal if the regulator only cares about consumer surplus.*

*Moreover, no MQS is beneficial when the regulator can control the information structure.*

Note that the optimal information structure is not unique in either case. The intuition is simple. Essentially the welfare maximizing regulator only cares about revealing ranking information to maximize matching efficiency, while the consumer-surplus-maximizing regulator prefers any information structures that reveal no information about ranking (so that the firms compete in an extreme Bertrand manner). The full revelation and full concealment tests are two benchmarks with the most straightforward disclosing rule and are invariant whether the test is restricted to be firm-independent or allowed to be correlated.[13] Since the only reason for imposing MQS in the

[13]Yang (2020) studies the consumer- and social-optimal information structures in the uncovered market model. Full concealment is still consumer-optimal among all information structures that reveals no ranking information. Full revelation, however, is no longer socially optimal. In fact the ranking-only policy is the unique socially optimal information structure due to the existence of demand effects.

main context is intensifying price competition through partial information, it does not bring further benefit if the test itself is under the regulator's control.

**Endogenous quality choice.** The benefits of the minimum quality standard in our model have a limited scope due to the assumption that the true qualities (both the distributions and the realizations) are exogenous. This simplification shuts down the channel of motivating the firms' quality provision, which could be an important justification for the use of MQS as documented in the existing literature. One interesting yet open question is how firms' endogenous quality choice interacts with a general test structure in the context of price competition, in which the information structure could be either controlled by the firms as in our model or regulated by the government in alternative settings.

Zapechelnyuk (2020) addresses such a question in a monopoly market, in which a regulator designs a certification rule (an information structure restricted to deterministic scores) to incentivize the firm's investment in deterministic quality provision. Shapiro (1986) studies the value of licensing/certification in a two-stage model but restricted to binary states and binary outputs. Oligopoly settings might introduce very different trade-offs, for instance, the tension between reducing differentiation and motivating investments. We leave the study of stochastic quality investment, general information structure and oligopoly markets for future works.

# Appendix A Proofs

*Proof of lemma 4.* We first characterize firm 1's interim payoff function $\Pi_1(s_1) = \int \pi_1 dG_2$ when firm 2 follows the proposed strategy $G_2 = G^\delta$. When $s_1 < s_0$, firm 1 is excluded and

$$\int_{\underline{q}}^{\bar{q}} \pi_1(s_1, s_2) dG_2(s_2) = 0$$

When $s_1 \in [s_0, s_0 + \delta)$, firm 1 acts as the monopolist when $s_2 < s_0 - \delta$, the high firm when $s_0 - \delta \leq s_2 < s_1$ and the low firm when $s_1 < s_2$, so

$$\begin{aligned}\Pi_1(s_1) = &\int_{\underline{q}}^{s_0-\delta} \pi^m(s_1) dG_2(s_2) + \int_{s_0-\delta}^{s_0+\delta} \pi_h^d(s_1, s_0) dG_2(s_2) + \int_{s_0+\delta}^{\bar{q}} \pi_l^d(s_2, s_1) dG_2(s_2) \\ =&F(s_0 - \delta)\pi^m(s_1) + [F(s_0 + \delta) - F(s_0 - \delta)]\pi_h^d(s_1, s_0) + \int_{s_0+\delta}^{\bar{q}} \pi_l^d(s_2, s_1) dF(s_2) \qquad (1)\end{aligned}$$

Note that this term is linear in $s_1$ since $\pi^m, \pi_h^d$ and $\pi_l^d$ are all linear in $s_1$. For simplicity we write it as $\Pi_1|_{[s_0, s_0+\delta)}(s_1) = ks_1 + n$.

When $s_1 \in [s_0 + \delta, \bar{q})$, firm 1 acts as the monopolist when $s_2 < s_0 - \delta$, the high firm when $s_0 - \delta \leq s_2 < s_1$ and the low firm when $s_1 < s_2$, so

$$\begin{aligned}\int_{\underline{q}}^{\bar{q}} \pi_1(s_1, s_2) dG_2(s_2) = &\int_{\underline{q}}^{s_0-\delta} \pi^m(s_1) dG_2(s_2) + \int_{s_0-\delta}^{s_0+\delta} \pi_h^d(s_1, s_0) dG_2(s_2) \\ &+ \int_{s_0+\delta}^{s_1} \pi_h^d(s_1, s_2) dG_2(s_2) + \int_{s_1}^{\bar{q}} \pi_l^d(s_2, s_1) dG_2(s_2) \qquad (2)\end{aligned}$$

It's straightforward to verify that this term is convex since the first two terms are linear in $s_1$ and the sum of the last two terms are quadratic in $s_1$. We write it as $\Pi_1|_{[s_0+\delta, \bar{q})}(s_1) = as_1^2 + bs_1 + c$.

In sum firm 1's interim payoff is

$$\Pi_1(s_1) = \mathbb{1}_{\{s_0 \leq s_1 < s_0+\delta\}}(ks_1 + n) + \mathbb{1}_{\{s_0+\delta \leq s_1 \leq \bar{q}\}}(as_1^2 + bs_1 + c),$$

where $(k, n, a, b, c)$ are constants determined by equation (1) and (2). We can verify that $\frac{\partial}{\partial s_1}\pi_-(s_0 + \delta) = \frac{\partial}{\partial s_1}\pi_+(s_0 + \delta)$ and $\frac{\partial}{\partial s_1}\pi$ is strictly increasing on $[s_0 + \delta, \bar{q}]$.

Let $(s', 0)$ denote the intersection point of the $x-$axis and the extension of $\Pi_1|_{[s_0, s_0+\delta)}$. We consider four scenarios: (1) $k > 0$ and $s' > s_0 - \delta$, (2) $k > 0$ and $s' = s_0 - \delta$, (3) $k > 0$ and $s' < s_0 - \delta$, and (4) $k \leq 0$ so $s'$ either does not exist or exceeds $\bar{q}$. The shape of $\Pi_1$ for the first 3 cases are illustrated figure 2.[14]

In the first and second case, we can verify that firm 1's best response is $G_1 = G^{\delta'}$ with $\delta' = s_0 - s'$ by invoking lemma 2. In particular, we let the support function $\phi(s_1)$ be the maximum of $\Pi_1(s_1)$ and the extension of $\Pi_1|_{[s_0, s_0+\delta)}$. It's straightforward to verify that

- $\phi$ is convex and $\phi \geq \Pi_1$,
- $\text{supp}G_1 = [\underline{q}, s'] \cup \{s_0\} \cup [2s_0 - s', \bar{q}] \subset \{s_1 : \phi(s_1) = \Pi_1(s_1)\}$,
- $E_F(\phi(s_1)) = E_{G^{\delta'}}(\phi(s_1))$ because $G^{\delta'} = F$ for all $s_1 \notin (s_0 - \delta', s_0 + \delta')$ and

$$E_F(\phi(s_1))|_{(s_0-\delta', s_0+\delta')} = F((s_0 - \delta', s_0 + \delta'))\phi(s_0) = E_G(\phi(s_1))|_{(s_0-\delta', s_0+\delta')}.$$

In the third case, firm 1's best response is $G_1 = G^{\delta'}$ in which $\delta'$ is uniquely determined as below. We can verify the optimality of $G^{\delta'}$ for each case in the same way as the first case.

1. when $s_0 > \dfrac{1}{2}(\bar{q} + \underline{q})$ and $2\Pi_1(s_0) > \Pi_1(\bar{q})$, $\delta' = \bar{q} - s_0$;
2. when $s_0 < \frac{1}{2}(\bar{q} + \underline{q})$ and $2\Pi_1(s_0) > \Pi_1(2s_0 - \underline{q})$, $\delta' = s_0 - \underline{q}$;
3. in all other cases $\delta < \min\{\bar{q} - s_0, s_0 - \underline{q}\}$ is the unique solution to

$$\frac{\Pi_1(s_0)}{s_0 - \tilde{\delta}}(s_1 - \tilde{\delta}) = \Pi_1(s_0 + \tilde{\delta}).$$

Finally we consider the case when $k \leq 0$. This scenario occurs only when $s_0$ is sufficiently small and hence $\delta$ is restricted to a small range $[\underline{q}, s_0]$. Firm 1's best response and the associated analysis is the same as the case when $s_0 < \frac{1}{2}(\bar{q} + \underline{q})$ and $2\Pi_1(s_0) > \Pi_1(2s_0 - \underline{q})$, that is, firm 1 best responds by choosing $G^{\delta'}$ such that $\delta' = s_0 - \underline{q}$. □

*Proof of lemma 5.* We first make the following observations:

[14] Although the graphs are generated from numerical settings with intermediate $s_0$ and interior $\delta$, the main features of general cases are captured. Moreover, the analysis is not restricted to the numerical example in the figure.

- Observation 1. Take the extension of the linear part $\Pi_1(s_1)|_{[s_0,s_0+\delta)} = ks_1 + n$ to $s_1 \in \mathbb{R}$ and denote it as $\Pi^{l-ext}(s_1|s_0,\delta)$. We observe $\lim_{\delta\to\underline{q}} \Pi^{l-ext}(\underline{q}|s_0,\delta) > 0 \ \forall \ s_0 > \underline{q}$. This can be shown by substituting $s_1$ in expression (1) with $\underline{q}$. The only negative term is the middle term $\pi_h^d(s_1 = \underline{\,}, s_0)$ since it is proportional to $(s_1 - s_0)$. When $\delta$ shrinks to 0, this term vanishes and the whole term is positive.

- Observation 2. For any $s_0 > \underline{q}$, the slope of the linear region $k$ is increasing continuously in $\delta$. Given (1), we can directly compute $\frac{\partial}{\partial\delta}k = \frac{2A^h - A^m}{\bar{q} - \underline{q}} = \frac{9\bar{q}^2 - 16\bar{q}\underline{q} + 8\underline{q}^2}{12(\bar{q} - \underline{q})^2} > 0$ since $\bar{q} > 2\underline{q}$.

- Observation 3. For any $s_0 > \underline{q}$, $\Pi_1(s_0)$ decreases continuously in $\delta$:
Given (1), we can directly compute $\frac{\partial}{\partial\delta}\Pi_1(s_0) = -\frac{1}{\bar{q} - \underline{q}}(A^m s_0 + A^l)\delta < 0$.

The first observation implies that for any $s_0$ we either have $k \le 0$ or $k > 0$ and $s' < \underline{q}$ and hence $s' < s_0 - \delta$. The second and the third observations together implies that as we increase $\delta$ from 0, $k$ drops. Either $k$ is remained to be non-positive before $\delta$ reaches the bound $\min\{s_0 - \underline{q}, \bar{q} - s_0\}$, or $k$ becomes positive after some cutoff and $s'$ increase continuously with $\delta$. Meanwhile, $s_0 - \delta$ decreases continuously with $\delta$, so as $\delta$ increases $s'$ and $s_0 - \delta$ cross at most once. Hence either one of the two (mutually exclusive) scenarios occur:

1. Interior equilibrium: if $s' = s_0 - \delta$ for some $\delta < \min\{s_0 - \underline{q}, \bar{q} - s_0\}$, this delta constitutes the symmetric equilibrium. See figure 3 panel (b) for a demonstration.

2. Corner equilibrium with $s_0 < s_0^l$: if either $k \le 0$ or $k > 0$ but $s' < s_0 - \delta$ when $\delta = s_0 - \underline{q} < \bar{q} - s_0$, we have a corner equilibrium such that $\delta' = \delta = \min\{s_0 - \underline{q}, \bar{q} - s_0\}$. The cutoff $s_0^l$ is determined by $s'(s_0 = s_0^l) = \underline{q}$. That this constitutes an equilibrium is established in lemma 4 and see figure 4 for a demonstration.

3. Corner equilibrium with $s_0 > s_0^u$: if $k \le 0$ or $k > 0$ but $s' < s_0 - \delta$ when $\delta = \min\{s_0 - \underline{q}, \bar{q} - s_0\}$, we have a corner equilibrium such that $\delta' = \delta = \min\{s_0 - \underline{q}, \bar{q} - s_0\}$. The cutoff $s_0^u$ is determined by $s'(s_0 = s_0^u) = 2\bar{q} - s_0^u$. That this constitutes an equilibrium is established in lemma 4 and see figure 4 for a demonstration.

Finally we finish the proof by showing that $s_0^l$ and $s_0^u$ are both unique. This follows directly from the three observations below:

- Observation 4. When $s_0 = \underline{q} + \epsilon$ or $s_0 \bar{q} - \epsilon$ for $\epsilon$ small enough, the equilibrium is corner.
- Observation 5. When $s_0 < \dfrac{\underline{q} + \bar{q}}{2}$, $s'$ increases in $s_0$ if we fix $\delta = s_0 - \underline{q}$, since $\Pi_1(s_0)$ decreases continuously in $s_0$ and $k$ increases continuously in $s_0$:
$\dfrac{\partial}{\partial s_0}\Pi_1(s_0)|_{\delta = s_0 - \underline{q}} = -\dfrac{A^l(\bar{q} - \underline{q} + 4s_0)}{\bar{q} - \underline{q}} < 0$, and $\dfrac{\partial}{\partial s_0}k|_{\delta = s_0 - \underline{q}} = \dfrac{2(A^h + A^l)}{\bar{q} - \underline{q}} > 0$.
- Observation 6. When $s_0 > \dfrac{\underline{q} + \bar{q}}{2}$, $s'$ decreases in $s_0$ when we fix $\delta = \bar{q} - s_0$, since $\Pi_1(s_0)$ increases continuously in $s_0$ and $k$ decreases continuously in $s_0$:
$\dfrac{\partial}{\partial s_0}\Pi_1(s_0)|_{\delta = \bar{q} - s_0} = \dfrac{A^m(4s_0 - \bar{q} - \underline{q})}{\bar{q} - \underline{q}} > 0$, and $\dfrac{\partial}{\partial s_0}k|_{\delta = \bar{q} - s_0} = \dfrac{2(A^m - A^h)}{\bar{q} - \underline{q}} \leq \frac{36\underline{q}^2}{49(\bar{q} - \underline{q})^2} < 0$.

Combining observation 4 and 5, there is a unique turning point from corner equilibrium $s' \leq \underline{q}$ to interior equilibrium $s' > \underline{q}$ if we increase $s_0$ from just above $\underline{q}$ and fix $\delta = s_0 - \underline{q}$. Similarly, there is a unique turning point from corner equilibrium $s' \leq 2s_0 - \bar{q}$ to interior equilibrium $s' > 2s_0 - \bar{q}$ if we decrease $s_0$ from just below $\bar{q}$ and fix $\delta = \bar{q} - s_0$.

□

*Proof of lemma 6.* Assume firm 2 is fully revealing its quality information, that is, $G_2 = F$, firm 1's interim expected profit is 0 when $s_1 < s_0$ and

$$\int \pi_1(s_1 = s_0, s_2)dF(s_2) = F(s_0)\pi_1^m(s_0) + \int_{s_0}^{\bar{q}} \pi_l(s_2, s_0)dF(s_2) > 0$$

when $s_1 = s_0$. $\Pi_1(s_1)$ is not convex, so fully revealing quality $G_1 = F$ is not a best response. □

*Proof of lemma 7.* We first prove that a local pooling strategy cannot arise in equilibrium.

Suppose firm 2 pools an interval $(a, b) \subset (s_0, \bar{q})$ at $\frac{a+b}{2}$. For any state $s \in (a, \frac{a+b}{2})$, firm 1's interim payoff is given by

$$\Pi_1(s) = \int_{\underline{q}}^{s_0} \pi^m(s)dG(s_2) + \int_{s_0}^{a} \pi_h^d(s, s_2)dG(s_2) + \int_{b}^{\bar{q}} \pi_l^d(s_2, s)dG(s_2) + \pi_l^d\left(\frac{a+b}{2}, s\right)[G(b) - G(a)].$$

For any state $s \in (\frac{a+b}{2}, b)$, firm 1's interim payoff is given by

$$\Pi_1(s) = \int_{\underline{q}}^{s_0} \pi^m(s)dG(s_2) + \int_{s_0}^{a} \pi_h^d(s, s_2)dG(s_2) + \int_{b}^{\bar{q}} \pi_l^d(s_2, s)dG(s_2) + \pi_h^d\left(s, \frac{a+b}{2}\right)[G(b) - G(a)]\,.$$

Note that $\Pi_1$ is linear in both cases and the slopes differ only in the last term. Since

$$\frac{d}{ds}\Pi_1|_{(a,\frac{a+b}{2})} - \frac{d}{ds}\Pi_1|_{(\frac{a+b}{2},b)} = -(A_l^d + A_h^d)\,[G(b) - G(a)] < 0,$$

$\Pi_1$ is piece-wise linear and overall convex on $(a, b)$. This implies that pooling at $(a, b)$ can not be a best response by firm 1.

That bi-pooling cannot arise in equilibrium for the same reason. If there is an interval $(a, b)$ such that in firm 2's strategy only two scores $c, d \in (a, b)$ are generated, we can easily show that the same strategy cannot be a best response by firm 1 since $\Pi_1$ is convex on $(a, b)$:

$$\frac{d}{ds}\Pi_1|_{(a,c)} < \frac{d}{ds}\Pi_1|_{(c,d)} < \frac{d}{ds}\Pi_1|_{(d,b)}.$$

□

*Proof of lemma 8.* The previous lemmas implies that $G$ must be continuous at any state $s > s_0 + \delta$ if $G$ involves pooling at $s_0$ with radius $\delta$ for any $\delta > 0$. When there is no such pooling, $G$ is continuous at all states above $s_0$.

By contradiction, assume there exists an interval $[a, b]$ such that $G$ is strictly increasing and $G \neq F$. Pick a state $s \in (a, b)$, we have

$$\begin{aligned}\Pi_1(s) = &\int_{\underline{q}}^{s_0} \pi^m(s)dG(s_2) + \int_{s_0}^{a} \pi^d(s, s_2)dG(s_2) + \int_{b}^{\bar{q}} \pi_l^d(s_2, s)dG(s_2) \\ &+ \int_{a}^{s} \pi_h^d(s, s_2)dG(s_2) + \int_{s}^{b} \pi_l^d(s_2, \tilde{s})dG(s_2)\end{aligned}$$

Note that the first three terms are linear in $s$. Since $G$ is continuous and hence almost everywhere differentiable, we can take the first order derivative of $\Pi$ at states where a pdf $g$ is well defined:

$$\frac{d}{dx}\Pi_1(s) = (A_h^d - A_l^d)G(s) - A_h^d G(a) - A_l^d G(b) + \tilde{A}$$

where $\tilde{A}$ is a constant. In order that $G$ constitute a "matching-pennies" type equilibrium, $\Pi_1(s)$ must be linear on $(a, b)$, implying a constant first order derivative. Hence $G(s)$ for $s \in (a, b)$ must be constant. There are two possibilities: (1) $(a, b)$ is contained in a pooling region, which contradicts with the fact that the only pooling occurs in a neighborhood of $s_0$ and $a > s_0 + \delta$; (2) $(a, b) \notin [s_0, s_0 + \delta)$ is contained in a super-interval $(a, b) \subset (c, d) \subset [s_0 + \delta, \bar{q}]$ such that $G$ is constant in $(c, d)$ and strictly increasing in $(c-\epsilon, c]$ and $[d, d+\epsilon')$ for some $\epsilon, \epsilon' > 0$. To show this is impossible, first note that it cannot be $G|_{[s_0+\delta,c]\cup[d,\bar{q}]} = F|_{[s_0+\delta,c]\cup[d,\bar{q}]}$ since it violates $G \in \text{MPC}(F)$. Hence we can repeat the "linear $\Pi$" argument for sub-intervals of $[s_0 + \delta, c] \cup [d, \bar{q}]$, leading to the conclusion that there can be no sub-intervals of $[s_0, \bar{q}]$ where $G$ is strictly increasing and $G \neq F$. This implies that either 1) $G(s) = G(s_0)$ for all $s \geq s_0$, 2) there exists a $\delta$ such that $G(s) = G(s_0)$ for all $s \in [s_0, s_0 + \delta]$ and $G(s) = F(s)$ for $s \geq s_0 + \delta$, or 3) $G$ is a step function with at least one jump points above $s_0$. The first two cases conform with the class of strategies $G^\delta$. The last case violates the lemma that there no mass point other than $s_0$ in any equilibrium.

□

*Proof of proposition 1.* When $\alpha = \frac{1}{2}$, the price is merely a transfer that does not affect the total welfare. Hence the regulator's objective is:

$$\begin{aligned}\max \sum_{i\in\{1,2\}} \int_\Theta \int_S \int_Q \theta q_i \mathbb{1}_{\{\text{purchase from firm } i\}} dG(q|s) dG(s) dH(\theta) \\ = \sum_{i\in\{1,2\}} \int_\Theta \int_S \theta s \mathbb{1}_{\{\text{purchase from firm } i\}} dG(s) dH(\theta)\end{aligned}$$

subject to the consumers' optimal product choice given equilibrium prices.

Note that when $s_0 = \underline{q}$, both firms fully reveal quality and the regulator can achieve its complete information outcome subject to the pricing equilibrium and the consumers' individual rationality. Any partial pooling leads to 1) fewer consumers purchasing from the high quality firm, and 2) lower matching efficiency since consumers randomize with equal probability when the realized qualities fall in the pooling region. □

*Proof of proposition 2.* In the duopoly market with perceived qualities $(s_l, s_h)$, the *ex post* consumer surplus is

$$CS^d(s_h, s_l) = \int_{\underline{\theta}}^{X} (\theta s_l - p_l) dH(\theta) + \int_{X}^{\bar{\theta}} (\theta s_h - p_h) dH(\theta) = C_h s_h + C_l s_l$$

where we denote $C_h = -\frac{11\underline{\theta}^2 - 14\bar{\theta}\underline{\theta} + 2\bar{\theta}^2}{18(\bar{\theta} - \underline{\theta})} > 0$, $C_l = \frac{11\bar{\theta}^2 - 14\bar{\theta}\underline{\theta} + 2\underline{\theta}^2}{18(\bar{\theta} - \underline{\theta})} > C_h$.

In the monopoly market with perceived quality $s$ the consumer surplus is

$$CS^m(s) = \int_{\bar{\theta}/2}^{\bar{\theta}} (\theta s - p^m(s)) dH(\theta) = \frac{1}{8} \frac{\bar{\theta}^2}{\bar{\theta} - \underline{\theta}} s = C_m s.$$

When $s_0 \leq s_0^l$, both firms choose $\delta = s_0 - \underline{q}$. The corresponding expected equilibrium consumer surplus is

$$\begin{aligned}
CS(s_0) =& F(s_0 + \delta)^2 CS^d(s_0, s_0) + 2F(s_0 + \delta) \int_{s_0+\delta}^{\bar{q}} CS^d(s, s_0) dF(s) + 2 \int_{s_0+\delta}^{\bar{q}} \int_{s_0+\delta}^{h} CS^d(h, l) dF(l) dF(h) \\
=& \frac{4s_0(s_0 - \underline{q})^2}{(\bar{q} - \underline{q})^2}(C_h + C_l) + \frac{4(s_0 - \underline{q})}{(\bar{q} - \underline{q})^2} \left[ C_h \frac{\bar{q}^2 - (2s_0 - \underline{q})^2}{2} + C_l s_0 (\bar{q} + \underline{q} - 2s_0) \right] \\
&+ \frac{2}{(\bar{q} - \underline{q})^2} \left[ (2C_h + C_l) \frac{\bar{q}^3 - (2s_0 - \underline{q})^3}{6} - C_h (2s_0 - \underline{q}) \frac{\bar{q}^2 - (2s_0 - \underline{q})^2}{2} - C_l \frac{(2s_0 - \underline{q})^2 (\bar{q} + \underline{q} - 2s_0)}{2} \right] \\
=& \frac{1}{(\bar{q} - \underline{q})^2} \left\{ 4s_0 (s_0 - \underline{q})^2 (C_h + C_l) - \underline{q} C_h (\bar{q}^2 - (2s_0 - \underline{q})^2) - C_l \underline{q}^2 (\bar{q} + \underline{q} - 2s_0) + \frac{(2C_h + C_l)(\bar{q}^3 - (2s_0 - \underline{q})^3)}{3} \right\}
\end{aligned}$$

Take the first order derivative we get

$$\begin{aligned}
CS'(s_0) =& \frac{1}{(\bar{q} - \underline{q})^2} \left\{ (C_h + C_l)(12s_0^2 - 16s_0\underline{q} + 4\underline{q}^2) + \underline{q} C_h (8s_0 - 4\underline{q}) + 2C_l \underline{q}^2 - 2(2C_h + C_l)(2s_0 - \underline{q})^2 \right\} \\
=& \frac{4}{(\bar{q} - \underline{q})^2} (C_l - C_h)(s_0 - \underline{q})^2 \geq 0,
\end{aligned}$$

so the consumer surplus increases with $s_0$ when $s_0 \leq s_0^l$. □

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