Information Design in Vertically Differentiated Oligopolies

Renkun Yang

Jinan University

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We conduct

a theoretical exercise that applies (competitive) Bayesian persuasion to a Shaked-Sutton duopoly (vertical diff. + hetero. consumer)

1. firms design own quality info and price after signal realization
2. a designer (literal or metaphorical) designs joint quality info and firms price after signal realization

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a theoretical exercise that applies (competitive) Bayesian persuasion to a Shaked-Sutton duopoly (vertical diff. + hetero. consumer)

1 firms design own quality info and price after signal realization

full revelation is dominant strategy **for some market structure**

& is Nash eqm **for some priors**

2 a designer (literal or metaphorical) designs joint quality info and firms price after signal realization

ranking maximizes social welfare

no-info maximizes consumer welfare

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a theoretical exercise that applies (competitive) Bayesian persuasion to a Shaked-Sutton duopoly (vertical diff. + hetero. consumer)

1. firms design own quality info and price after signal realization

   Hwang, Kim & Boleslavsky (2019) for **horizontal** diff.
   & price **before** realization

2. a designer (literal or metaphorical) designs joint quality info and firms price after signal realization

   Armstrong & Zhou (2022) for **horizontal** diff.
   & reduced to **1-dim**

Model

Market environment

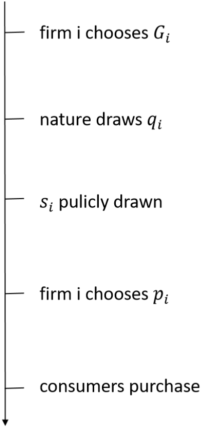
- 2 firms, qualities $q_i \sim F(\cdot)$ i.i.d.
- unit mass consumers, taste $\theta \sim H = U[\underline{\theta}, \bar{\theta}]$

  utility: $u = \begin{cases} \theta q_i - p_i & \text{purchase from firm } i \\ 0 & \text{no purchase} \end{cases}$

Information strategy

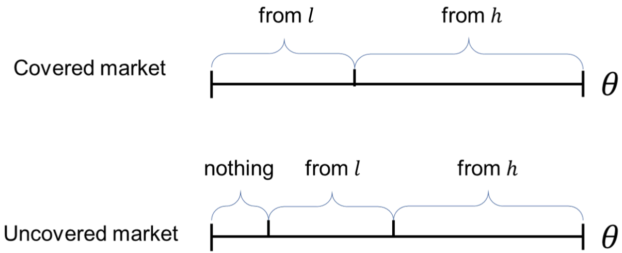
- info structure (M_i, τ_i), $\tau_i : Q_i \to \Delta(M_i)$
- equiv., a distribution $G_i \in MPC(F)$ over posterior means s_i (perceived quality)

Timing



(Nonexhaustive) market specification

The price eqm is sorting. Depending on $H(\theta)$, the market might be covered or uncovered:



Covered Market

Price equilibrium: fixed market share

Assumption (covered market). $\dfrac{\bar{q} - \underline{q}}{\underline{q}} \leq \dfrac{3\underline{\theta}}{\bar{\theta} - 2\underline{\theta}}$.

Market shares are fixed: type cutoff $= \frac{1}{3}(\bar{\theta} + \underline{\theta})$

$$p_h^* = \frac{(2\bar{\theta} - \underline{\theta})}{3}(s_h - s_l), \; p_l^* = \frac{(\bar{\theta} - 2\underline{\theta})}{3}(s_h - s_l).$$

$$\pi_h^* = \frac{(2\bar{\theta} - \underline{\theta})^2}{9(\bar{\theta} - \underline{\theta})}(s_h - s_l), \; \pi_l^* = \frac{(\bar{\theta} - 2\underline{\theta})^2}{9(\bar{\theta} - \underline{\theta})}(s_h - s_l).$$

Full info: dominant strategy

Proposition

In covered duopoly, full revelation is a dominant strategy equilibrium.

Intuition

- Information strategies have no market share effect
- The price effect is unambiguous:
  - more information $\Rightarrow$ more dispersion in “perceived” quality
  - larger perceived differentiation $\Rightarrow$ less price competition

Full info: dominant strategy

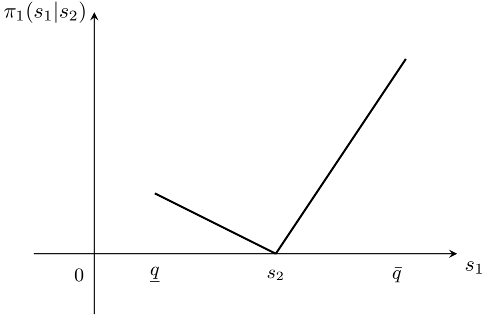


Figure 1: Firm 1's profit function $\pi_1(s_1|s_2)$ given a fixed s_2.

Full info: dominant strategy

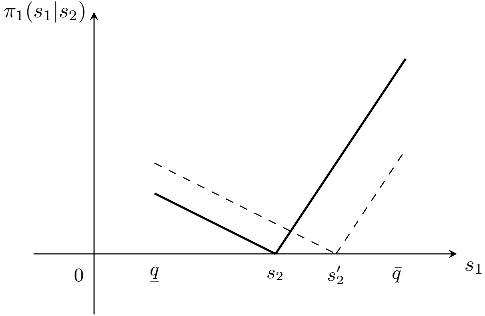


Figure 1: Firm 1's profit function $\pi_1(s_1|s_2)$ given a fixed s_2.

Uncovered Market

Price equilibrium: signal-dependent market share

Assumption (uncovered market). $\underline{\theta} = 0, \bar{\theta} = 1$.

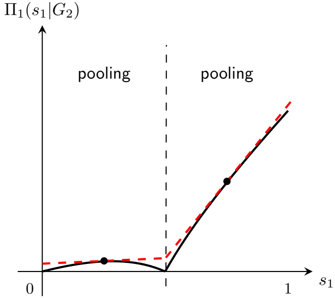
Equilibrium market shares depend on prices (and scores):

$$p_l^*(s_h, s_l) = \frac{s_l(s_h - s_l)}{4s_h - s_l}, \qquad p_h^*(s_h, s_l) = \frac{2s_h(s_h - s_l)}{4s_h - s_l}$$

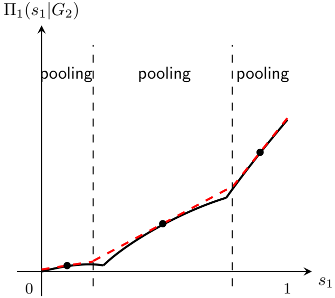
$$\pi_l^*(s_h, s_l) = \frac{s_l s_h(s_h - s_l)}{(4s_h - s_l)^2}, \qquad \pi_h^*(s_h, s_l) = \frac{4s_h^2(s_h - s_l)}{(4s_h - s_l)^2}$$

Full info: not dominant

Firm 2 conceals, firm 1 induces 1 cutoff



Firm 2 induces 1 cutoff, firm 1 induces 2



Intuition

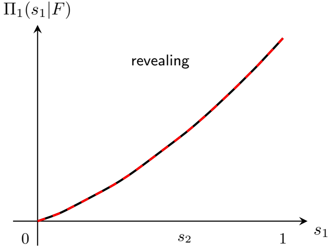
The information strategies affect market outcome through:

- (unilateral) price effect: reveal more to soften competition
- market share effect: reveal less to attract consumers
- (strategic) price effect: reveal more to avoid Bertrand competition

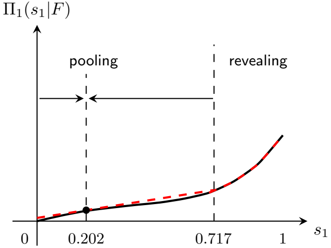
Full info: Nash? Not always!

Full revelation equilibrium does not always obtain

If firm 2 reveals $F(x) = x^{0.5}$



If firm 2 reveals $F(x) = x^3$



Full info NE: a sufficient condition

Corollary (uniform prior)

If $\bar{q} \leq 2.5\underline{q}$, full revelation constitutes a symm eqm.

Rough Intuition

scope of differentiation vs. consumer attraction

- when $\underline{q} = 0$, firm 1 attracts no consumer at $s_1 = \underline{q}$: pooling upwards
- when $\underline{q}$ is close to $\bar{q}$, firm 1 attracts enough consumers even at the bottom: enlarge differentiation

Otherwise?

Proposition (Nonexistence)

If full revelation is not an eqm, then there exists no symmetric eqm.

Lemma (Characterization)

If a symmetric equilibrium exists (but not full revelation), it must be such that (1) G^* is continuous and strictly increasing almost everywhere, (2) $\Pi_i(s_i|G^*)$ is weakly convex, and (3) $\Pi_i(s_i|G^*)$ is strictly convex whenever $G^*(s_i) = F(s_i)$ and linear whenever $G^*(s_i) \neq F(s_i)$.

*linearity characterizes the partial-info eqm in HBK (19)

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Social- and Consumer-Optimum

Proposition

- social optimal info: nothing-but-ranking;
- consumer optimal info: anything-but-ranking, e.g., full concealment.

Intuition

- social optimal: ranking for sorting, no more for market coverage
- consumer optimal: no-ranking for 0-price Bertrand

*qualitatively similar to AZ (22) but simpler

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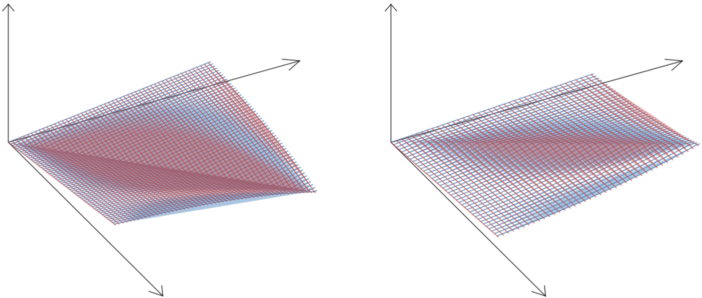

Figure 2: Left panel: *ex post* social welfare; right panel: *ex post* consumer surplus

Related Literature

Vertical differentiation and quality disclosure

- Shaked & Sutton, 1982; Board, 2009; Levin et al., 2009

(Optimal/competitive) information design

- Methods: Kamenica & Gentzkow, 2011, 2016; Dworczak & Maritini, 2019; Kleiner et al., 2020; Arieli et al., 2020
- Applications: Gill and Sgroi, 2012; Roesler & Szentes, 2017; Zapechelnyuk, 2020; Condorelli & Szentes, 2021; Hwang, Kim & Boleslavsky, 2019; Armstrong & Zhou, 2022

Thanks!