Information Design in Vertically Differentiated Oligopolies

Renkun Yang

Jinan University

August 8, 2022
We conduct

a theoretical exercise that applies (competitive) Bayesian persuasion to a Shaked-Sutton duopoly (vertical diff. + hetero. consumer)

1. firms design own quality info and price after signal realization

2. a designer (literal or metaphorical) designs joint quality info and firms price after signal realization
We conduct a theoretical exercise that applies (competitive) Bayesian persuasion to a Shaked-Sutton duopoly (vertical diff. + hetero. consumer).

1. **firms** design own quality info and price after signal realization
   - full revelation is dominant strategy for some market structure & is Nash eqm for some priors

2. a **designer** (literal or metaphorical) designs joint quality info and firms price after signal realization
   - ranking maximizes social welfare
   - no-info maximizes consumer welfare
We conduct a theoretical exercise that applies (competitive) Bayesian persuasion to a Shaked-Sutton duopoly (vertical diff. + hetero. consumer)

1. **Firms** design **own quality info** and **price after signal realization**

   Hwang, Kim & Boleslavsky (2019) for **horizontal** diff. & price **before** realization

2. **A designer** (literal or metaphorical) designs **joint quality info** and **firms price after signal realization**

   Armstrong & Zhou (2022) for **horizontal** diff. & reduced to **1-dim**
**Model**

### Market environment

- 2 firms, qualities $q_i \sim F(\cdot)$ i.i.d.
- unit mass consumers, taste $\theta \sim H = U[\theta, \bar{\theta}]$

utility: \[ u = \begin{cases} \theta q_i - p_i & \text{purchase from firm } i \\ 0 & \text{no purchase} \end{cases} \]

### Information strategy

- info structure $(M_i, \tau_i)$, $\tau_i : Q_i \to \Delta(M_i)$
- equiv., a distribution $G_i \in MPC(F)$ over posterior means $s_i$ (perceived quality)

### Timing

- firm i chooses $G_i$
- nature draws $q_i$
- $s_i$ publicly drawn
- firm i chooses $p_i$
- consumers purchase
The price eqm is sorting. Depending on $H(\theta)$, the market might be covered or uncovered:
Covered Market
Price equilibrium: fixed market share

Assumption (covered market).

\[ \frac{\bar{q} - q}{q} \leq \frac{3\theta}{\theta - 2\bar{\theta}}. \]

Market shares are fixed: type cutoff = \( \frac{1}{3}(\bar{\theta} + \theta) \)

\[
\begin{align*}
    p^*_h &= \frac{(2\bar{\theta} - \theta)}{3}(s_h - s_l), \\
    p^*_l &= \frac{(\bar{\theta} - 2\theta)}{3}(s_h - s_l). \\
    \pi^*_h &= \frac{(2\bar{\theta} - \theta)^2}{9(\theta - \theta)}(s_h - s_l), \\
    \pi^*_l &= \frac{(\bar{\theta} - 2\theta)^2}{9(\theta - \theta)}(s_h - s_l).
\end{align*}
\]
Proposition

In covered duopoly, full revelation is a dominant strategy equilibrium.

Intuition

- Information strategies have no market share effect
- The price effect is unambiguous:
  - more information $\Rightarrow$ more dispersion in “perceived” quality
  - larger perceived differentiation $\Rightarrow$ less price competition
Figure 1: Firm 1’s profit function $\pi_1(s_1|s_2)$ given a fixed $s_2$. 
Full info: dominant strategy

Figure 1: Firm 1’s profit function $\pi_1(s_1|s_2)$ given a fixed $s_2$. 
Uncovered Market
Assumption (uncovered market). $\theta = 0, \bar{\theta} = 1$.

Equilibrium market shares depend on prices (and scores):

\[
\begin{align*}
  p_l^*(s_h, s_l) &= \frac{s_l(s_h - s_l)}{4s_h - s_l}, \\
  \pi_l^*(s_h, s_l) &= \frac{s_l s_h (s_h - s_l)}{(4s_h - s_l)^2}, \\
  p_h^*(s_h, s_l) &= \frac{2s_h(s_h - s_l)}{4s_h - s_l}, \\
  \pi_h^*(s_h, s_l) &= \frac{4s_h^2(s_h - s_l)}{(4s_h - s_l)^2}.
\end{align*}
\]
Firm 2 conceals, firm 1 induces 1 cutoff

\[ \Pi_1(s_1|G_2) \]

pooling

Firm 2 induces 1 cutoff, firm 1 induces 2

\[ \Pi_1(s_1|G_2) \]

pooling
The information strategies affect market outcome through:

- (unilateral) price effect: reveal more to soften competition
- market share effect: reveal less to attract consumers
- (strategic) price effect: reveal more to avoid Bertrand competition
Full info: Nash? Not always!

Full revelation equilibrium does not always obtain

If firm 2 reveals $F(x) = x^{0.5}$

If firm 2 reveals $F(x) = x^3$
Corollary (uniform prior)

If $\bar{q} \leq 2.5q$, full revelation constitutes a symm eqm.

Rough Intuition

scope of differentiation vs. consumer attraction

- when $q = 0$, firm 1 attracts no consumer at $s_1 = q$: pooling upwards
- when $q$ is close to $\bar{q}$, firm 1 attracts enough consumers even at the bottom: enlarge differentiation
Proposition (Nonexistence)

If full revelation is not an eqm, then there exists no symmetric eqm.

Lemma (Characterization)

If a symmetric equilibrium exists (but not full revelation), it must be such that (1) $G^*$ is continuous and strictly increasing almost everywhere, (2) $\Pi_i(s_i|G^*)$ is weakly convex, and (3) $\Pi_i(s_i|G^*)$ is strictly convex whenever $G^*(s_i) = F(s_i)$ and linear whenever $G^*(s_i) \neq F(s_i)$.

*linearity characterizes the partial-info eqm in HBK (19)
Proposition (Nonexistence)

If full revelation is not an eqm, then there exists no symmetric eqm.

Lemma (Characterization)

If a symmetric equilibrium exists (but not full revelation), it must be such that (1) $G^*$ is continuous and strictly increasing almost everywhere, (2) $\Pi_i(s_i|G^*)$ is weakly convex, and (3) $\Pi_i(s_i|G^*)$ is strictly convex whenever $G^*(s_i) = F(s_i)$ and linear whenever $G^*(s_i) \neq F(s_i)$.

*linearity characterizes the partial-info eqm in HBK (19)
Proposition

- social optimal info: nothing-but-ranking;
- consumer optimal info: anything-but-ranking, e.g., full concealment.

Intuition

- social optimal: ranking for sorting, no more for market coverage
- consumer optimal: no-ranking for 0-price Bertrand

*qualitatively similar to AZ (22) but simpler*
Proposition

- social optimal info: nothing-but-ranking;
- consumer optimal info: anything-but-ranking, e.g., full concealment.

Intuition

- social optimal: ranking for sorting, no more for market coverage
- consumer optimal: no-ranking for 0-price Bertrand

*qualitatively similar to AZ (22) but simpler
Social- and Consumer-optimum

Figure 2: Left panel: *ex post* social welfare; right panel: *ex post* consumer surplus
Related Literature

**Vertical differentiation and quality disclosure**

- Shaked & Sutton, 1982; Board, 2009; Levin et al., 2009

**(Optimal/competitive) information design**

- Methods: Kamenica & Gentzkow, 2011, 2016; Dworczak & Maritini, 2019; Kleiner et al., 2020; Arieli et al., 2020

Thanks!