The Termination Clause as a Sequential Screening Device^{*}

Renkun Yang

Department of Economics

The Ohio State University

yang.3180@osu.edu

Abstract

In a takeover setting in which buyers arrive sequentially and the value uncertainty is resolved over time, we show that a termination clause can serve as a device for the target firm to sequentially screen the initial buyer. The initial buyer with an optimistic estimate accepts a contract with a high floor price and high termination fee to mitigate future competition; a pessimistic initial buyer, in contrast, chooses to stay flexible by accepting a low price and low termination fee. We characterize the revenue-maximizing contract and provide a rationale for the use of go-shop provisions in the M&A market.

JEL Classification Numbers: D42, D44, D82, D86

Keywords: Dynamic mechanism design; termination fee; price discrimination; go-shop provision

^{*}I thank Yaron Azrieli, Huiyi Guo, Paul J. Healy, Jone Jae Lee, Jim Peck, and Lixin Ye for their helpful comments. For their helpful discussions I would also like to thank conference participants in the 2019 China Meeting of Econometric Society, the 2019 Midwest Theory Conference in Bloomington, the 2019 annual meeting of Midwest Economic Association in St. Louis, and the 2019 Stony Brook International Conference on Game Theory.

1 Introduction

Termination clauses are widely adopted in real world markets including takeover auctions, property sales or leases, and labor contracts. One prominent yet theoretically understudied practice is the "go-shop" provision, a deal-making tactic in the mergers and acquisition (hereafter M&A) market that has emerged and grown in popularity since the private equity boom of 2004-2007.¹ In contrast to a typical (no-shop) negotiation, in which the target firm cannot solicit alternative proposals after signing the acquisition agreement, a go-shop provision allows the seller to actively solicit potential bidders in a post-signing go-shop period of 30-45 days. The agreement typically includes a floor price the initial buyer is willing to pay and a termination fee compensated to this initial bidder in case of a break-up. For instance, *Topps*, a 2007 merger agreement between the Michael Eisner-led private equity and Topps Co., a manufacturer of baseball cards, is a prominent decision about the use of go-shop period of 40 days after the signing date and a termination fee that amounted to 3% of the transaction value. Topp's financial advisor contacted 107 potential bidders, yet only one bidder, Upper Deck, made a serious proposal. Nonetheless, the competing proposal was not regarded as superior and the deal was finally completed with Eisner.², ³

When it was introduced, this corporate sale method precipitated controversies among legal scholars, financial practitioners and economists. These controversies were largely a response to the *Revlon* duty principle, which, as stated in the Delaware Supreme Court's 1986 decision about the *Revlon* case, declares that during the sale of a company, the board must work to ensure that the the stockholders' benefit is maximized. There are two major opinions about the practice of go-shop and window-shop clauses.

¹Although it is commonly believed that go-shop provisions formally arose around 2004 (Houtman and Morton, 2007; Subramanian, 2007; Wang, 2018), some key elements of this deal format existed for longer. See Sautter (2007) for a detailed discussion of the emergence of go-shop deals and their relationship to other acquisition mechanisms, such as no-shop or window-shop deals.

²Of course, it is not always the case that a go-shop deal is won by the initial acquirer. In fact, in the samples examined by Subramanian (2007), Antoniades et al. (2016) and Gogineni and Puthenpurackal (2017), a higher bidder appears 10%-13% of the time. For instance, the sale of CKE Restaurants in 2009 was completed with an alternative bidder, Apollo Management, instead of the initial acquirer, Thomas H. Lee, after the post-signing market canvassing. Thomas H. Lee was paid a termination fee.

³The *Topps* case, together with *Lear* (the merger agreement between Lear and Carl Icahn that was issued by the Vice Chancellor of Delaware court one day after *Topps*), was commonly viewed as an endorsement of go-shop provisions by the court. See Sautter (2007), Houtman and Morton (2007) and Subramanian and Zhao (2019) for institutional details.

First, those who hold the managerial discretion opinion argue that to secure deals with friendly bidders, incumbent managers use termination fee provisions to create disadvantage and deter the entry of potential third parties.⁴ Although the contract formally allows the target to actively search for higher bidders (a provision that appears to be pro-competitive), favoritism towards the initial buyer reveals that the go-shop clause is simply "window-dressing".⁵

Second, a theory based on entry efficiency suggests that the termination provision could lead to more competition. Wang (2018), for instance, presents a model that features interdependent value and costly information acquisition. The termination fee serves as the compensation to the first buyer for incurring the cost and providing information externality to subsequent competitors. Through this channel the go-shop provision could arise as an optimal dynamic mechanism and revenue-dominate a static auction due to higher level of entry. Empirical evidence seems to support the pro-competitive view. Analysts have found that go-shop deals usually are associated with higher premiums, effectively attract additional competitors and improve deal completion rates (Subramanian, 2007; Jeon and Lee, 2014; Gogineni and Puthenpurackal, 2017).⁶ Given these evidence, the managerial discretion theory that suggests the anti-competitive nature is not fully plausible.

This paper examines a different and novel channel, sequential screening, through which the go-shop provision could fulfill the *Revlon* duty. By explicitly modeling the due diligence procedure as an information release stage, we capture the interaction of buyer learning and the timing of contracts, an important insight that is understudied in the literature on M&A deals. We consider a stylized two-period model. In the first period, the target firm provides a menu of go-shop contracts to an initial buyer who holds a noisy signal about his true valuation. In the second period, an additional bidder is solicited and invited to a modified English auction, where both bidders learn their true value prior to bidding. The go-shop agreement specifies a floor price and a termination fee. The floor price binds the first buyer and serves as the reserve price for the second buyer. The termination fee is essentially paid by the new buyer in case the initial buyer loses the deal.

 $^{^{4}}$ See Kahan and Klausner (1995) for an explanation of the theory that through this favor exchange managers can facilitate job security.

⁵See, for example, an article by Andrew Sorkin titled "Go Shop Can Be a Fig Leaf for a Deal to Hide Behind" published in The New York Times on February 28, 2011. https://dealbook.nytimes.com/2011/02/28/go-shop-can-be-a-fig-leaf-for-a-deal-to-hide-behind.

⁶These findings are also consistent with earlier studies that focus on similar pre-competition contracts that include break-up fees (Bates and Lemmon, 2003; Officer, 2003; Boone and Mulherin, 2007). There is, however, one exception. Antoniades et al. (2016) find a negative correlation between go-shop deals and initial offer premium and no evidence for final premium enhancement.

We find that the go-shop contract facilitates surplus extraction and improves seller's revenue because of the following forces. First, the *ex ante* contracting allows the seller to extract surplus in expectation. This extraction is implemented through the promised floor price, which works in a fashion comparable to the admission fees/upfront payments in the literature (e.g., McAfee and McMillan, 1987; Courty and Li, 2000). Moreover, the floor price in the go-shop deal is even higher than under expectation pricing in an one-to-one negotiation, because the compensation in the event of termination improves the option value for the initial buyer. Second, of greater interest is the fact that the go-shop contract can be made incentive compatible and screen the early buyer's initial type. The intuition resembles the standard single crossing argument: the initial buyer with an optimistic signal values the target more, and, hence, it holds a stronger intensity of preference over the probability of winning. A go-shop clause with a higher floor price and termination fee creates a larger bidding advantage that secures for the initial buyer a higher chance of winning. Thus, a more optimistic (pessimistic) buyer is willing to choose a higher (lower) contract from the menu. Such a separating result further facilitates revenue extraction through inter-temporal price discrimination. Third, the go-shop deal still allows future competition in case a late-arrived buyer possesses a high value. Such an *ex post* adjustment further increases revenue through efficiency enhancement and rent extraction from the entrant.

The rest of this paper proceeds as follows. The subsection examines the relevant literature. Section 2 introduces the model setting. Section 3 characterizes the seller's optimal go-shop mechanism. Section 4 elaborates the mechanism as well as the revenue performance of the optimal go-shop provision by comparing to two benchmarks: the optimal static and dynamic optimal mechanisms. Section 5 concludes.

1.1 Related Literature

Wang (2018), to the best of the author's knowledge, offers the only other formal model of the goshop provision. Her insight of informational externality and entry efficiency is in line with earlier discussions of sequential negotiations (e.g., Fishman, 1988; Bulow and Klemperer, 2009; Roberts and Sweeting, 2013) and with studies of preemptive bidding and other deal protection devices that resolves free-riding problems (e.g., Grossman and Hart, 1980; Berkovitch et al., 1989; Berkovitch and Khanna, 1990). Our rationale complements the work of Wang (2018) in the respect that we consider the same deal format, but in a different perspective: we consider an independent private value environment (hence without informational externality) and focus on dynamic incentives, without explicit modeling costly entry, while Wang (2018) focuses on entry in the form of costly information acquisition of a common value component.

By adopting a more general information environment, this paper contributes to the literature on strategic *ex ante* contracts (Aghion and Bolton, 1987; Hua, 2007; Che and Lewis, 2007; Choi, 2009; Hua, 2012; Dimopoulos and Sacchetto, 2014, etc.). We consider a flexible information structure for the initial bidder, who observes an *ex ante* signal in the first stage, of which the precision can be made arbitrary. In comparison, early studies, such as Aghion and Bolton (1987) consider publicly known value for the incumbent. Che and Lewis (2007) assume the initial bidder is fully informed from the beginning, while Hua (2007) assumes that the initial bidder has no *ex ante* information. Each represents an extreme case of our information structure. We show that the main insights of these studies carry over in our environment despite the different clauses analyzed: the *ex ante* contract can be used as a strategic tool to (partially) deter entry and extract rent from entrants.

The general information environment allows us to link the strategic contract literature and the dynamic mechanism design literature (Courty and Li, 2000; Eső and Szentes, 2007; Krähmer and Strausz, 2015; Akan et al., 2015; Deb and Said, 2015; Pavan et al., 2014; Lu and Ye, 2019, etc.). Our model differs from these in the respect that we assume a restricted form of dynamic arrival process,⁷ and more importantly, we study a specific form of mechanism, the go-shop provision, rather than general direct mechanisms. Thus, our model is in line with Nocke et al. (2011) that study advanced discount purchases and Ely et al. (2017) that study overbooking in flight tickets market, both of which identify real world market practices that take restricted formats but still resemble the key idea of sequential screening.

Finally, this paper is broadly related to the literature on search deterrence (Armstrong and Zhou, 2015; Lu and Wang, 2019; Li and Zhao, 2019), although in our model the seller, the side without private information, has the outside option to search for. Nevertheless, our optimal go-shop contract can be interpreted as "self search deterrence": in order to collect higher revenue, the seller

 $^{^{7}}$ For a more general treatment of mechanism design with stochastic and strategic dynamic arrivals, see Garrett (2016), Garrett (2017), etc.

partially deters her own search for outside option (in a strategic sense) by distorting the allocation in favor of the initial buyer.

2 Model

We consider a two-period model in which a seller (S) is selling an indivisible good at zero cost to two privately informed buyers. Both parties are risk neutral and neither discounts the future.

In period one, the initial buyer (B1) arrives and observes partial information about his willingnessto-pay for the good. We denote this information the *initial type* $\tau \in \mathbf{T} = [\underline{\tau}, \overline{\tau}]$ with cumulative distribution function (CDF) $F(\tau)$ and associated density (PDF) $f(\tau)$.

In period two, a new buyer (B2) shows up after the seller's solicitation. Both buyers learn their private valuations, $v_1, v_2 \in [v, \bar{v}]$. B2's value v_2 is distributed according to CDF $H(\cdot)$. B1's value v_1 follows distribution $G_{\tau}(\cdot)$, conditional on his initial signal τ . Moreover we denote unconditional distribution $G(\cdot)$ as B1's prior value distribution. Throughout the paper we assume $G_{\tau}(v_1), H(v_2)$ are continuously differentiable with positive density $g_{\tau}(v_1)$ and $h(v_2)$, respectively. The distributions G_{τ} have the monotone likelihood ratio property (MLRP); thus a higher initial signal implies a more favorable estimate of the *ex post* value for B1.

Assumption 1 (MLRP). τ is ordered by MLRP: $\frac{g_{\tau}(v)}{g_{\tau'}(v)} \ge \frac{g_{\tau}(v')}{g_{\tau'}(v')}$ for any $v \ge v'$ and $\tau \ge \tau'$.

For the sake of analytical simplicity, we make throughout two additional regularity assumptions that are in line with the static and dynamic mechanism design literature. Assumption 2 is the standard Myerson regularity that avoids any ironing procedure in the optimal auction design. Assumption 3 generalizes this condition to the dynamic version where B1 possesses initial information τ . Both assumptions ensure the monotonicity of the optimal contract.

Assumption 2 (Static Regularity). $v - \frac{1-G(v)}{g(v)}$ and $v - \frac{1-H(v)}{h(v)}$ are both increasing in v. Assumption 3 (Dynamic Regularity). $v + \frac{1-F(\tau)}{f(\tau)} \frac{\partial G_{\tau}(v)/\partial \tau}{g_{\tau}(v)}$ is increasing in both τ and v.

We restrict our attention to the go-shop mechanism specified as a last stage auction augmented by a pre-auction contract. In the first period, the seller offers B1 a menu of contracts, each consisting of a "floor price" \underline{b} and a "termination fee" f. B1 can choose one contract from the menu $(\underline{b}(\tau), f(\tau))_{\tau \in \mathbb{T}}$ or simply stay out of the mechanism. If B1 accepts one contract (\underline{b}, f) , he agrees to bid no less than \underline{b} in the subsequent auction. In the second period, both buyers are invited to a modified English auction with reserve price \underline{b} . When B2 wins (the dropping out price $b_2 > b_1$) he pays $b_1 + f$ and B1 is compensated with f. Otherwise, B1 wins and pays $\max\{b_2, \underline{b}\}$. The timing of the model is summarized in Figure 1.

t = 1	t = 1.5	$\begin{array}{c} t=2 \\ \end{array}$	t = 2.5
• B1 observes τ .	 S announces a menu (<u>b</u>(τ), f(τ))_τ; B1 accepts one contract or opts out. 	 S announces the signed contract; B1 learns v₁; B2 arrives, learns v₂. 	 If B1 accepts one contract, S invites both bidders to the auction; If B1 opts out, S sells to B2 at a posted price.

Figure 1: Time-line of a go-shop provision

Before proceeding to the next section we briefly discuss our model assumptions.

Arrival and information process. We model a stylized arrival process similar to those described by Hua (2007) and Che and Lewis (2007), where two bidders arrive sequentially. The existence of an initial buyer is prevalent in the corporate takeover context, especially in buyer-initiated deals in which, prior to being contacted by an initial acquirer, targets have not necessarily considered themselves to be for sale.⁸ In these cases, potential competitors might not even be aware that a deal is possible prior to the targets' post-check solicitation. Even in seller-initiated deals, targets often contact only a small number of prominent or favored candidate acquirers before a broader market canvassing starts.⁹

We assume that before signing the contract the first buyer has an imperfect signal regarding his final valuation. By varying the informativeness of this signal we can bridge two extreme cases: Hua (2007), in which the initial buyer has no initial information; and Che and Lewis (2007), in which each buyer is fully informed upon arrival.¹⁰ After the due diligence phase, with certain

⁸This paper mainly focuses on the dynamic incentive issue in the go-shop deals and, hence, it does not explicitly model the deal initiation process. The strategic initiation of an acquisition deal could be an interesting topic to pursue. See Gorbenko and Malenko (2018).

 $^{^{9}}$ A possible situation is the buyout deal, in which the targets' managerial boards have (partial) control or a share of the initial acquirer.

 $^{^{10}}$ As noted in section 3 and 4, a neutrality result regarding break-up fees in Che and Lewis (2007) no longer holds in our general setting while Hua (2007) resembles a special case of the optimal direct mechanism in our model.

confidentiality agreements the target discloses additional non-public information to the bidders. Because the main focus of our model is the target's strategic contracting with the initial bidder, we assume for the sake of simplicity that the second bidder knows his value upon arrival. This assumption is justified because the targets are typically required to provide the initial and solicited buyers with the same information and confidentiality agreements.¹¹

Finally, although our model contains only two buyers, it covers the case in which there are multiple new bidders. This is because i) we allow for asymmetric (prior) value distributions across bidders and ii) we model the competition as a modified English auction so that the bidding strategy does not depend on the number of bidders. When there are N-1 new bidders with $v_i \sim H_i(\cdot) \forall i \in \{2, ..., N\}$, it is equivalent to consider only one new bidder whose value is the order statistic $v_{(n)} = \max\{v_2, ..., v_N\}$.

Competition format and contract transparency. We model the competition stage as an English auction for two reasons. On the practical side, the English auction is one of the most commonly used selling mechanism in the M&A markets.¹² Moreover, it captures a prevalent practice in go-shop provisions, wherein initial bidders have the right to match any proposal made by new competitors. Without "matching rights" the competition becomes implicit and takes the form of sequential search.¹³

On the theoretical side, we choose the English auction because in our model the bidders necessarily hold asymmetric value distribution in the bidding stage. The English auction admits a unique weakly dominant strategy equilibrium that is advantageous in that it allows us to avoid the technical complexity of other bidding models, such as asymmetric first price auctions. Furthermore, the dominant strategy equilibrium implies that it does not matter whether or not the seller makes the first stage contract public. Hence, we do not need to consider the disclosure decision by the seller at the beginning of period two.

¹¹See Houtman and Morton (2007) for details.

 $^{^{12}}$ The auction format used in mergers and acquisitions is generally complicated and context specific. Nonetheless, M&A deals usually include multiple rounds of bidding and take the form of "pay-second-highest-bid," so that the English auction is a reasonable theoretical approximation. See also Gorbenko and Malenko (2014) and Wang (2018) for discussions of this issue.

 $^{^{13}\}mathrm{See}$ Wang (2011) for a search model with termination terms in a labor market context.

3 Main Results

We solve for the optimal go-shop contract in the backward order. Section 3.1 characterizes the bidding equilibrium. The equilibrium payoffs for B1 determine his relevant *incentive compatibility* (IC) and *individual rationality* (IR) constraints when selecting contracts in the first stage. Section 3.2 solves the seller's problem subject to the IC and IR constraints. We show that the optimal go-shop mechanism equates the seller's marginal revenue from the better informed B1 and the last-minute shopper B2, and it is characterized by the dynamic virtual valuations of each bidder.

3.1 Bidding Equilibrium

Suppose B1 has accepted contract (\underline{b}, f) in stage 1. At the beginning of stage 2, both bidders participate in the modified English auction. B1 starts with the first bid \underline{b} and the price ascends over time. Both bidders decide whether to drop out at each instance. Since the values are independent across bidders, the bidding game is strategically equivalent to a modified second price auction wherein each bidder chooses his drop-out price. As in a standard second price auction, in the unique dominant strategy equilibrium, both bidders bid their "essential" values subject to B1's floor price constraint. In this environment B1's opportunity cost of winning is f when losing; thus, he values the object at $v_1 - f$. B2, on the other hand, shades his bid by f because he has to pay an additional amount of f when he wins. The following lemma formally states this truthful bidding equilibrium.

Lemma 3.1 (Bidding Equilibrium). Suppose bidder one has accepted the first period contract (\underline{b}, f) . There exists a unique weakly dominant strategy equilibrium wherein the bidders drop out at prices:

$$b_1 = \max\{v_1 - f, \underline{b}\}, \ b_2 = v_2 - f \quad \forall v_1, v_2$$

The proof is standard and thus omitted. Figure 2 illustrates the *ex post* equilibrium allocation in the space of value realization. The allocation is efficient when bidder 2's value is above $\underline{b} + f$; otherwise the allocation is distorted in favor of bidder 1 exclusively. Henceforth, for both expositional and analytical simplicity, we let $t = \underline{b} + f$ denote this "deterrence price" and transform the problem of designing an optimal menu of contracts $\{(\underline{b}(\tau), f(\tau))\}_{\tau}$ to the equivalent problem of

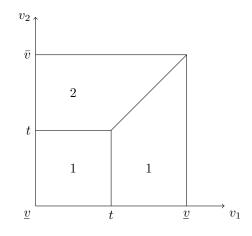


Figure 2: Equilibrium allocation in the bidding stage

designing $\{(t(\tau), f(\tau))\}_{\tau}$.

Given the equilibrium we can compute B1's payoff given contract (t, f) and value $v_1 = v$:

$$\begin{split} u_1^G(v,t,f) &= \begin{cases} (v-\underline{b})H(t) + [1-H(t)]f & \text{if } v < t \\ \int_{\underline{v}}^t (v-\underline{b})dH(x) + \int_t^v (v-x+f)dH(x) + \int_v^{\overline{v}} f dH(x) & \text{if } v \ge t \end{cases} \\ &= \begin{cases} f + (v-t)H(t) & \text{if } v < t \\ f + \int_t^v H(x)dx & \text{if } v \ge t \end{cases} \end{split}$$

Thus, B1's expected equilibrium payoff when he is of initial type τ and accepts contract (t, f) is given by:

$$U_1(\tau, t, f) = \int_{\underline{v}}^{\overline{v}} \pi_1(v, t, f) dG_\tau(v)$$

= $f + \int_t^{\overline{v}} H(v) \left[1 - G_\tau(v)\right] dv - H(t) \int_{\underline{v}}^t G_\tau(v) dv$

Note that payoff U_1 is additively separable in f and t. In brief, we write $U_1(\tau, t, f) = f + B(\tau, t)$.

3.2 Optimal Go-Shop Provision

In this section we show that a version of Envelope theorem holds that converts the IC constraints to a monotonicity condition combined with an integral formula of B1's interim payoff. This allows us to focus on the class of *incentive compatible* contracts and solve the seller's problem in the standard first order approach, as in the screening literature. Let $U_1(\tau) = U_1(\tau, t(\tau), f(\tau))$ be B1's expected truth-telling payoff. The following proposition states this result explicitly.

Proposition 3.1 (Incentive Compatibility). A go-shop provision is incentive compatible if and only if the following holds:

- $t(\tau)$ is increasing in τ ,
- The envelope theorem holds: $U'_1(\tau) = \frac{\partial B(\tau, t(\tau))}{\partial \tau}.$

Before proceeding to the next step, we provide some intuition about the incentives and tradeoffs that B1 faces. In the present model B1 needs to trade off a higher deterrence price against a high termination fee. Here the deterrence price t directly determines the expected allocation, while the expected transfer payment is jointly determined by both t and f. Given the MLRP assumption, the higher initial type τ possesses higher estimates of the final value v_1 , and, hence, it has a higher marginal value for allocation, while all types have the same constant marginal value for money. Such heterogeneity in marginal value resembles the single-crossing property in a standard selling mechanism, and naturally it provides the seller with room to design an incentive compatible mechanism. In particular, an optimistic type (high τ) would like to accept a contract with a higher deterrence price to "secure" the item, even though it is associated with a higher floor price; a pessimistic type (low τ), on the other hand, prefers more flexibility to compete for the item only when the *ex post* value realization is sufficiently high.

The next proposition reduces the set of IR constraints to that of the lowest type.

Proposition 3.2 (Individual Rationality). An incentive compatible go-shop provision is individual rational if and only if $f(\underline{\tau}) + B(\underline{\tau}, t(\underline{\tau})) \ge 0$.

The seller's problem is to maximize the total revenue, subject to the incentive and participation

constraints. The revenue collected given each τ is:

$$R(\tau) = \underbrace{(t(\tau) - f(\tau))[H(t(\tau)) + G_{\tau}(t(\tau)) - H(t(\tau))G_{\tau}(t(\tau))]}_{\text{revenue when } \underline{b} \text{ sets price}} + \underbrace{\int_{t(\tau)}^{\bar{v}} \int_{t(\tau)}^{v_1} (v_2 - f(\tau))dH(v_2)dG_{\tau}(v_1)}_{\text{revenue when 1 wins and } b_2 \text{ sets price}} + \underbrace{\int_{t(\tau)}^{\bar{v}} \int_{v_1}^{\bar{v}} (v_1 - f(\tau))dH(v_2)dG_{\tau}(v_1)}_{\text{revenue when 2 wins and } b_1 \text{ sets price}} = t(\tau)C(\tau, t(\tau)) + D(\tau, t(\tau)) - f(\tau)$$

where

$$C(\tau, t(\tau)) = H(t(\tau)) + G_{\tau}(t(\tau)) - H(t(\tau))G_{\tau}(t_{\tau}(\tau)),$$
$$D(\tau, t(\tau)) = \int_{t(\tau)}^{\bar{v}} v \left[1 - G_{\tau}(v)\right] dH(v) + \int_{t(\tau)}^{\bar{v}} v \left[1 - H(v)\right] dG_{\tau}(v)$$

Given the characterization of IC and IR constraints, we can rewrite the seller's problem as summarized in the following proposition:

Proposition 3.3. The solution to the following program solves the original seller's problem.

$$\max_{t(\tau)} R_G = \int_{\underline{\tau}}^{\overline{\tau}} \left[t(\tau)C(\tau, t(\tau)) + D(\tau, t(\tau)) + B(\tau, t(\tau)) - \int_{\underline{\tau}}^{\tau} \frac{\partial}{\partial x} B(x, t(x)) dx \right] dF(\tau)$$
(1)

s.t.
$$t(\tau)$$
 is non-decreasing. (2)

To solve the problem listed above we first ignore (2) and solve the relaxed program and then verify the validity of the monotonicity constraint under assumption 1 and 2. Since the *ex ante* expected revenue term is additive separable in τ , the relaxed program can be solved by pointwise maximization. The following proposition characterizes the optimal deterrence price t, which, combined with the binding IR constraint and Envelope theorem, uniquely determines the optimal menu of go-shop contracts.

Proposition 3.4 (Optimal Go-Shop Provision: Characterization). The optimal go-shop provision

exists and satisfies the following necessary conditions:

$$E_{G(\cdot|\tau)}\left[\phi_1(v_1,\tau)|v_1 < t(\tau)\right] = t(\tau) - \frac{1 - H(t(\tau))}{h(t(\tau))}$$
(3)

$$f(\tau) + B(\tau, t(\tau)) = \int_{\underline{\tau}}^{\tau} \frac{\partial B(x, t(x))}{\partial x} dx$$
(4)

where $\phi_1(v_1, \tau)$ is the dynamic virtual valuation of B1:

$$\phi_1(v_1,\tau) = v_1 + \frac{1 - F(\tau)}{f(\tau)} \frac{\partial G(v_1|\tau)/\partial \tau}{g(v_1|\tau)}$$

The optimal solution $t(\tau)$ is monotone increasing in τ under assumption 3.

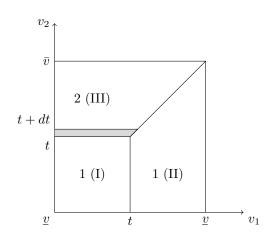


Figure 3: Equilibrium allocation in the bidding stage

Equation (3) represents the first order condition and explicitly elaborates the trade-off the seller faces when deciding the deterrence price t. The left-hand side is B1's truncated expected virtual valuation conditional on his type being below the deterrence price, which represents the expected marginal contribution by B1. The right-hand side is B2's virtual valuation evaluated at $v_2 = t$, as in the standard auction design literature. The underlying intuition can be illustrated in Figure 3. Specifically, we assume a marginal increase dt in the deterrence price t. Such a change leads to the following first-order effect: there is a shift of allocation from B2 to B1 (represented by the shaded area) with magnitude h(t)G(t). That is, on one hand, there is a marginal increase in the probability for B1 to secure the item. The seller can extract surplus from this additional proportion of B1 by the amount of $E_G(v|v \leq t)$ through expectation charges in floor prices. On the other hand, this revenue gain is at the expense of a marginal loss of the new buyer. This marginal B2 has value tand could have contributed virtual value $t - \frac{1-H(t)}{h(t)}$ to the total revenue. The optimal mechanism involves a deterrence price that equalizes the marginal benefit and marginal cost, as expressed in (3). Consider the extreme scenario where B1 does not value the item at all, i.e., $v_1 = 0$. In this case, the seller does not value B1 and the optimal go-shop provision simply reduces to an optimal selling schedule to B2, which is exactly the posted price mechanism defined by $p - \frac{1-H(p)}{h(p)} = 0$. At the other extreme, where $v_1 = \bar{v}$, B1 is known to hold the highest possible value. The seller will directly sell the item to buyer 1 at \bar{v} and does not consider buyer 2, corresponding to the case in which buyer 2 is fully deterred by $t = \bar{v}$ and f = 0.

Remark. Notice that $\phi_1(v_1)$, which we obtained in Proposition 3.4, is in the form of dynamic virtual valuation, which is also found (but used differently) in the optimal dynamic mechanism design literature (Courty and Li, 2000; Eső and Szentes, 2007; Pavan et al., 2014). This "coincidence" is not surprising: the virtual valuation terms, either dynamic $\phi(\cdot)$ or static $\psi(\cdot)$, are intrinsic statistics of the information structures for any incentive compatible mechanisms and are independent of the detailed mechanism formats.

Remark. Although we assume static and dynamic regularity for analytical elegance, they are not needed for the go-shop provision to work. See appendix for examples in which the monotonicity of the virtual valuation terms is not satisfied yet our analysis still works.

To further understand the implication of proposition 3.4, we present in the following corollaries two extreme cases regarding the information content of the initial signal.

Corollary 3.1 (No *ex ante* Information). *When B1 possesses no initial information, the optimal deterrence price satisfies:*

$$E_G(v|v < t) = t - \frac{1 - H(t)}{h(t)}.$$
(5)

Corollary 3.1 presents the condition for optimal deterrence price t when B1's initial signal is uninformative. In this case, B1 signs the contract before he observes any private information, and so the seller could appropriate all his information rent through the contracting procedure. Hence, the revenue contribution is reduced to simply his valuation v_1 , and the marginal revenue collected from B1 at deterrence price t is the truncated expected value of bidder 1, whose value is distributed in the shaded area in Figure 3.

Corollary 3.2 (No *ex post* Information). *When B1 possesses no ex post information, the optimal deterrence price satisfies:*

$$v - \frac{1 - G(v)}{g(v)} = t(v) - \frac{1 - H(t(v))}{h(t(v))}.$$
(6)

Furthermore, $t(\tau) \equiv t(v) = v$ when G = H.

Corollary 3.2 considers the other extreme, in which B1's initial signal is fully informative. In fact when B1 has no additional information in the post-contract stage, the environment is essentially static. The dynamic virtual valuation reduces to the static version. When the value distributions are symmetric, equation (6) further implies that the seller will fully elicit the first bidder's value in the contract stage and set the deterrence price to this level, which means that the equilibrium allocation is always efficient.

The following corollary summarizes the role of go-shop negotiation from the perspective of the literature on strategic *ex ante* contracts.

Corollary 3.3 (Contract as a barrier to entry and rent extraction). In general, the first stage agreement in the optimal go-shop provision can be used as a deterrence to potential competitors.

- The optimal go-shop negotiation yields a larger rate of deal completion than an optimal auction, but may cause misallocation between the two bidders;
- The deterrence price increases as H increases in the monotone hazard rate ordering;
- The surplus of the rival bidder is always weakly lower in the optimal go-shop provision than in an efficient auction.

The first statement directly follows from the bidding equilibrium and the resulting equilibrium allocation presented in section 4.1. The initial bidder, by signing the *ex ante* contract, obtains a bidding advantage in the competition stage. This is in line with the general finding that contracts

can be used as strategic tools to deter entry,¹⁴ as in the literature initiated by Aghion and Bolton (1987). The second statement follows proposition 3.4. It essentially states that the "entry barrier" increases as the level of competition increases, which could be induced by a better value distribution (in the statistical sense) or an increase in the number of potential competitors. The third statement claims that including the pre-contract in the go-shop deal always reduces the new buyer's surplus, which is in line with the findings of Hua (2007) and Choi (2009), despite the fact that they focus on different formats of strategic contracts with a favored bidder. Note that if we pay attention to the extreme case in which the initial buyer has no new information, then the third statement reduces to corollary 1 in Che and Lewis (2007), which states that "the non-recipient buyer is unharmed by the breakup fee." Our result implies that this is true only in the knife-edge case without considering B1's learning during the due diligence phase.

4 Revenue Performance

To better understand the revenue performance of the optimal go-shop provision, we compare it to two theoretical benchmarks. Section 4.1 considers the optimal static mechanism where the seller ignores B1's initial type and proposes a direct mechanism until both bidders are present. Section 4.2 considers the optimal dynamic mechanism, wherein the seller can fully specify a dynamic mechanism without any restriction.

4.1 Optimal static mechanism

First consider the static benchmark wherein the seller waits until the second period and proposes a static mechanism. In this situation, B1's initial information makes no difference; it is as if the seller were selling to two buyers with value distribution G and H in a static model. As is well known, the optimal mechanism allocates the item to the bidder who has the highest and positive virtual valuation (Myerson, 1981). This can be implemented by an auction with an adjusted winning rule and bidder-specific reserve prices.

¹⁴In our paper the entry deterrence is partial because the new buyer can still submit an effective bid if his value is sufficiently high. Furthermore, we do not explicitly model costly entry, and so B2 always (artificially) participates in the auction stage. However, the deterrence still takes an implicit form in that B2 can make no effective offer as long as $v_2 < t$.

In general, it is difficult to obtain a closed form revenue comparison between the optimal go-shop provision and the optimal static auction because of the complexity in both mechanisms.¹⁵ Instead, we elaborate the underlying distinction in terms of the seller's "efficiency vs. rent extraction" trade-off, compare the static mechanism and the two extremes of go-shop mechanisms (one with a fully informative initial signal and one with no initial information), and use a parametrized family of distributions to draw conclusions about the general case.

Consider two (potentially sub-optimal) benchmarks in which i) the go-shop provision involves an identical contract (t, f) (so there is still intertemporal pricing but no first-stage screening) and ii) the seller runs a symmetric auction with an identical reserve price r in period 2. The revenue terms are given as follows:

$$R_{SG} = t[H(t) + G(t) - H(t)G(t)] + \int_{t}^{\bar{v}} \int_{t}^{v_{1}} v_{2}dH(v_{2})dG(v_{1}) + \int_{t}^{\bar{v}} \int_{v_{1}}^{\bar{v}} v_{1}dH(v_{2})dG(v_{1}) - f$$
$$R_{SS} = r[H(r) + G(r) - 2H(r)G(r)] + \int_{r}^{\bar{v}} \int_{r}^{v_{1}} v_{2}dH(v_{2})dG(v_{1}) + \int_{r}^{\bar{v}} \int_{v_{1}}^{\bar{v}} v_{1}dH(v_{2})dG(v_{1})$$

where the termination fee is determined by the participation constraint of B1's lowest type.

We can clearly see that as written in the current form, how the deterrence price t and the reserve price r work is similar, except that: 1) the go-shop provision sells the item at price t with quantity G(t)H(t) when both bidders are below the reserve price, while the static mechanism induces no sale in the corresponding region; and 2) the go-shop provision compensates the early bidder with a lump-sum transfer f to maintain the "reserve" price t. This comparison reflects an underlying efficiency-rent trade-off: the go-shop provision induces a higher probability of sale while incurring a cost of f.

In general environments these two restricted formats are both sub-optimal within each class of mechanisms. They are, however, optimal in some circumstances in which we are able to quantify the revenue comparison. Specifically, the go-shop provision with a uniform t is optimal when there is no initial information, while the symmetric auction with a uniform r is optimal when the environment is symmetric G = H. We explicitly compare the revenue when both hold:

Proposition 4.1 (Revenue comparison: no *ex ante* Information). When τ is uninformative, the

 $^{^{15}\}mathrm{A}$ closed-form revenue comparison is available when both the type and value distributions are binary for all bidders. See appendix for the analytical results.

optimal go-shop negotiation always generates higher revenue than the optimal symmetric auction. If G = H, the optimal go-shop negotiation generates higher revenue than any optimal static mechanism.

The revenue comparison in case G = H directly follows because the optimal symmetric auction implements the optimal static mechanism. The go-shop provision can generate more revenue by increasing the sale probability at a lower cost whenever the seller can extract surplus via the interim rather than *ex post* participation constraint.

The underlying intuition can be illustrated as follows. Compare the go-shop provision with a (potentially sub-optimal) deterrence price $t = r^*$ where r^* represents the Myersonian reserve price. In Figure 3, the seller generates the same revenue from region (II) and (III) in the two mechanisms, while in the go-shop provision the seller gains additional revenue r^* from region (I) but pays a lump-sum f in all regions. B1's binding participation constraint essentially states that

payment from (I) + payment from (II) = v_1 from (I) + v_1 from (II) + f,

which implies (payment from $(I) - f \ge 0$) because B1's net payoff in region (II) is guaranteed to be positive.

Intuitively, in the no-initial-information extreme the seller is able to extract all surplus from buyer 1, which always outweighs the cost of promised compensation. This illustrates, too, that as the initial information becomes more precise, the revenue advantage of go-shop negotiation should vanish because the seller has to compromise more and more information rents to B1 to attain truthful revelation. The next proposition shows that the revenue comparison between the two mechanisms reverts at the other extreme, when B1 obtains full information in period 1.

Proposition 4.2 (Revenue comparison: no *ex post* Information). When τ is fully informative and G = H, the optimal go-shop negotiation is revenue equivalent to a static auction without reserve price, which always generates a weakly lower revenue than the optimal static mechanism.

The reversion of the revenue ranking is also intuitive. The main source of revenue superiority of any dynamic mechanism, the exploitation of interim instead of *ex post* IR constraints, vanishes to zero when the initial signal becomes fully informative. Meanwhile, the intrinsic "must-sell" restriction of the go-shop mechanism remains the same and, hence, it bounds the revenue away from the optimal level. In particular, a go-shop mechanism can at most be revenue equivalent to an ordinary auction without reserve price, which is the optimal static mechanism under the must-sell constraints.

The continuity of the revenue and optimal policy functions directly implies that the go-shop provision generates higher revenue when B1 faces a sufficient level of uncertainty about the final value before due diligence. The following proposition and corollary confirm this intuition by quantifying the information content of τ using a parametrized family of conditional distribution $G(v|\tau, a)$.

Proposition 4.3. Assume the prior is symmetric G = H, the conditional distribution $G_{\tau}(v)$ is parametrized by $a \in A \subset \mathbb{R}$, written as $G(v|\tau, a)$, and $\frac{\partial^2 G(v|\tau, a)}{\partial \tau \partial a} \leq 0 \quad \forall \tau, a, v$. Then the revenue generated from the optimal go-shop provision decreases in a.

Corollary 4.1. Assume the prior is symmetric G = H and $\frac{\partial^2 G(v|\tau, a)}{\partial \tau \partial a} \leq 0 \ \forall \tau, a, v$. There exists an a^* such that $R_G \geq R_S$ if and only if $a \leq a^*$

The parametrized family of conditional distribution, although seemingly restrictive, does not impose too great of a conceptual burden. The parameter a is simply an indicator of the informativeness of initial type τ . When a approaches its minimum value, the information structure converges to the extreme case in which B1 has no ex ante information. When a approaches its maximum, it converges to the other extreme, which is essentially a static environment. The sufficient (but not necessary) condition in the proposition simply imposes a structured quantification to the idea that "the informativeness of τ increases as a increases." In the examples below we present two commonly used information structures to illustrate the parametrization and the revenue comparison.

Example 4.1 (Multiplicative Linear Experiment). Let G(v) = H(v) = v on [0,1], $F(\tau) = \tau$ on [0,1], and $G(v|\tau,a) = a(2\tau - 1)v^2 - (2a\tau - a - 1)v$ on [0,1] where $a \in [0,1]$. It is easy to verify that this family of distribution satisfies the sufficient condition $\frac{\partial^2 G(v|\tau,a)}{\partial \tau \partial a} \leq 0 \forall \tau, a, v$. Hence, the revenue from the optimal go-shop provision is decreasing in a. However, the nature of this information structure imposes an upper bound on a that prevents it from approaching the "no ex post information" extreme (when a > 1, the formula no longer admits a valid cumulative distribution function). We can verify numerically that the optimal go-shop provision is revenue superior to the static Myerson mechanism for all $a \in [0, 1]$. **Example 4.2** (Truncated Normal Distribution). Let the prior and the signals be both truncated Normal over [0,1]. In particular, assume $v \sim N(\frac{1}{2},1;0,1)$, $and\tau|v \sim N(v,\frac{1}{a})$ so that $v|\tau \sim N(\mu,\sigma^2;0,1)$ where $\mu = \frac{\frac{1}{2}+a\tau}{1+a}$, $\sigma = \sqrt{\frac{a}{1+a}}$ and we refer to $a = \frac{1}{\sigma^2}$ as the signal precision. This family could approach both extremes by setting $a \to 0$ and $a \to \infty$. Although it does not satisfy the sufficient condition $\frac{\partial^2 G(v|\tau,a)}{\partial \tau \partial a} \leq 0 \ \forall \tau, a, v$, we can numerically verify that the revenue monotonicity of the optimal go-shop provision still holds.

4.2 Optimal dynamic mechanism

In this section we consider the fully optimal dynamic mechanism. We adopt the method developed in the dynamic mechanism design literature (Courty and Li, 2000; Eső and Szentes, 2007; Pavan et al., 2014, etc.) and find the optimal direct mechanism using the Myersonian approach of "local IC characterization" under regularity conditions.

Since the dynamic revelation principle applies, we focus on the following direct mechanism without loss of generality. B1 reports his type $\hat{\tau}$ in period 1 and both bidders report their values \hat{v}_1 and \hat{v}_2 in period 2. At the end of period 2, each bidder *i* is allocated the good with probability $q_i(\hat{\tau}, \hat{v}_1, \hat{v}_2)$ and each makes a payment $t_i(\hat{\tau}, \hat{v}_1, \hat{v}_2)$ to the seller. The optimal dynamic mechanism allocates the item to the bidder with the highest non-negative virtual valuation:

Proposition 4.4 (Optimal direct mechanism, see Pavan et al. (2014)). In an optimal direct mechanism that is incentive compatible and individually rational, the allocation rule is determined as follows:

$$q_1(\tau, v_1, v_2) = \begin{cases} 1 & \text{if } \phi_1(\tau, v_1) > \max\{0, \psi_2(v_2)\} \\ 0 & \text{otherwise} \end{cases}$$
$$q_2(\tau, v_1, v_2) = \begin{cases} 1 & \text{if } \psi_2(v_2) > \max\{0, \phi_1(\tau, v_1)\} \\ 0 & \text{otherwise} \end{cases}.$$

The transfer rule is determined by the Envelope theorem and the individual rationality constraints (IR of $\underline{\tau}$ -type B1 and \underline{v} -type B2). A full characterization can be found in, for instance, Pavan et al. (2014). The following corollary provides a simple indirect implementation in a modified handicap auction under some distributional restrictions.

Corollary 4.2. If the impulse response function $\frac{\partial G/\partial \tau}{g}$ does not depend on the realized value v_1 , the optimal mechanism can be implemented by a modified handicap auction:

- In stage 1, the first buyer can acquire a premium p_1 at an up-front fee c_1 .
- In stage 2, two buyers participate in a second price auction where the winner pays the loser's bid plus his own premium (where bidder 2's premium is zero).

Note that the optimal go-shop mechanism also involves the same virtual valuation term, although it does so without fully appropriating the power of rent extraction through dynamic mechanisms. More specifically, the go-shop provision cannot implement the fully optimum because of three restrictions. First, the floor price and the termination fee clause is pre-determined. Hence, the only way it can affect the allocation in the auction stage is through the deterrence price t, which sets a bar for B2 to effectively participate. This tool is not responsive to the second stage bids, while the optimal direct mechanism is fully contingent (in a direct mechanism, the second stage reports correspond to the second stage bids in its auction implementation). Second, in the optimal goshop negotiation, the deterrence price t cannot make full use of the interim participation constraint because the allocation rule and the payment rule are inter-correlated. More specifically, the floor price $\underline{b} = t - f$ simultaneously affects the deterrence price t and the payment when B1 wins. However, in a handicap auction the premium is related to the upfront payment only through the incentive constraints. Third, the fact that the target firm is always sold limits the capability of surplus extraction.¹⁶ We elaborate these restrictions in the following numerical example.

For the ease of expression and visualization, we consider an example in which B1's initial signal follows a binary distribution¹⁷ and the value distributions G_{τ} are within the family of power distributions: $\tau = h$ with probability α and $\tau = l$ with probability $1 - \alpha$. The *h* type has a value distribution $G_h(v) = v^2$ on [0, 1] and the *l* type B1 as well as B2 have a uniform value distribution $G_l(v) = H(v) = v$ on [0, 1]. Figure 4 presents the equilibrium allocation by the mechanism and

 $^{^{16}}$ This problem can be potentially resolved by introducing a reverse termination fee, which allows the initial bidder to terminate the deal at a certain transfer if the *ex post* realization is low. This practice is also common in real deals (although less frequent than the target termination fee), but this is not the main focus of the current paper.

 $^{1^{7}}$ Such a binary-signal-continuous-value setting is qualitatively similar to the general model. We only need to adjust the dynamic virtual valuation term to its discrete version. The full analytical results are provided in the online appendix.

the initial type of B1. We can see that the optimal go-shop provision captures the main feature of sequential screening in the optimal direct mechanism; that is, the final allocation is biased toward B1, and the high type obtains more advantage. This heterogenous bias elaborates the intuition of "locking": the optimistic B1 values the item more and, hence, he prefers to secure the allocation by admitting a higher deterrence price. The distinction is also clear. The allocation border is restricted to a horizontal cutoff t combined with the 45 degree line in the go-shop deal, but it is more flexible in the fully optimal dynamic mechanism.

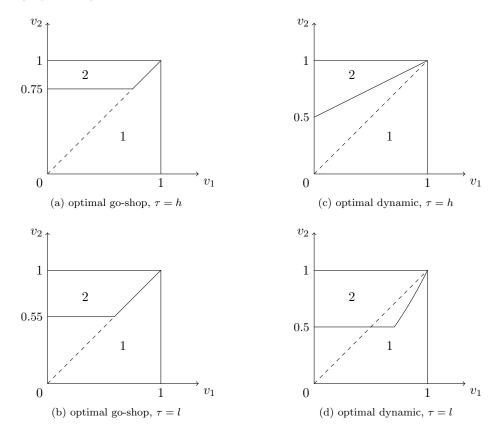


Figure 4: ex post allocation with binary type and power value distribution

Figure 5 reports numerical results of revenue comparison between the optimal go-shop provision, optimal static auction, and optimal dynamic mechanism when τ is binary. The left panel considers the underlying value distributions $G_h(v) = v^a$, $G_l(v) = v^{\frac{1}{a}}$, H(v) = v over [0, 1] and $a \in [1, \infty)$. The solid blue curve represents the expected revenue of the optimal go-shop provision as a function of a, which captures the precision of the initial signal of B1. The dashed (dotted) red curve corresponds

to the expected revenue of the optimal dynamic (static) mechanism. When a = 1, the initial signal is non-informative. When $a \to \infty$, the initial signal approaches the full-information limit. We can see from the graph that when a is small the optimal go-shop provision has a large revenue advantage compared to the static auction and achieves a surprisingly high proportion of the secondbest revenue collected in the optimal dynamic mechanism. As a increases, the go-shop provision becomes out-performed by the optimal static auction. Intuitively, the high type B1 becomes very likely to hold value 1 while the low type becomes almost surely valueless as a increases. In this situation, the go-shop provision (modeled in the current format) is heavily constrained by the "must-sell" nature: in case of a low type B1 (such that $v_1 = 0$ and B1 never wins), a "must-sell" seller cannot impose any non-trivial reserve price to the new bidder for rent extraction.

In the right panel, we present a different parametric setting wherein $G_h(v) = v^b$, $G_l(v) = H(v) = v$ over [0, 1] and $b \in [1, \infty)$. In this case the initial bidder has on average a more favorable value distribution than the new bidder and the low-type's value distribution is bounded from below. Since the informativeness of the initial type is limited by nature, the go-shop provision performs substantially well. Numerically it attains more than 95% of the revenue collected in a fully optimal mechanism.

Remark. Although the go-shop provision could under-perform in the full information extreme, as illustrated in the left panel, it is less a concern in practice. On one hand, the limits mainly come from the "must-sell" constraint, which can potentially be overcome by incorporating the reverse termination fee. On the other hand, it is in practice more plausible to consider a bounded (from below) value distribution even for the least optimistic initial buyer, because the initial buyer in real go-shop deals are usually one of, if not the most interested acquirer.

5 Conclusion

By identifying a potential application of sequential screening in the real world, this paper contributes to the literature of dynamic mechanism design. We show that the go-shop provision, a selling mechanism in the M&A market that contains break-up clauses and allows the seller to actively solicit additional potential buyers, could be revenue-superior to traditional static auctions.

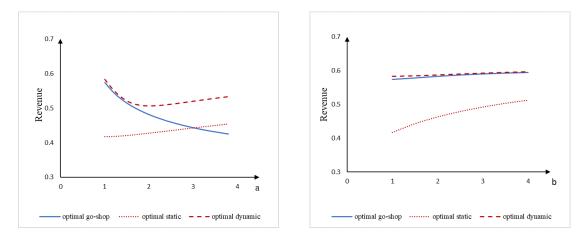


Figure 5: Revenue comparison with binary initial type, $\alpha = \frac{1}{2}$, and (a) $G_h(v) = v^a, G_l(v) = v^{1/a}, H(v) = v$; (b) $G_h(v) = v^b, G_l(v) = H(v) = v$

Complementing the existing rationale based on costly entry for the use of termination fees, our theory presents a different channel of revenue enhancement. In a key feature, the initial acquirer holds *ex ante* beliefs and learns additional information about the final value during the due diligence phase. By using a contingent contract that contains a menu of "floor-price-termination-fee" combinations, the seller can extract more rents from the newly solicited buyers by giving the initial buyer a bidding advantage, and the seller can appropriate the information rent from the initial buyer in both the *ex ante* and the *ex post* stages. An initial buyer who has higher estimates is willing to "lock" the item at a higher price that is associated with a higher compensation in case of a deal breakup. A more pessimistic initial buyer, however, only promises a low minimum price and delays the additional payment until the new information arrives. In this way, the pessimistic buyer purchases only if the *ex post* value turns out to be high or the competitor turns out to be weak.

We study a stylized model of selling mechanisms with break-up clauses. Of course we cannot take all interesting features of the go-shop deals, or more generally the M&A markets, into consideration. Here we list a few of them that could be considered in future studies. First, an important feature of the M&A market is the potential agency problems. In contrast to product markets, in the acquisition market the managerial board that conducts the sale usually has misaligned interests with the share-holders, which serves as the base for the managerial discretion theory that argues against the pro-competitive effect of the go-shop clauses. Our model abstracts this issue away and focus exclusively on the market structure. A natural extension is to investigate the use of deal protection devices in a three-parties (owner, agent, and buyer) setting. Second, we follow the contract design approach in this paper and, hence, implicitly assume that the seller has full bargaining power. In real deals, however, buyers negotiate actively with the seller over acquisition agreements, rather than choose passively from a menu of contracts offered by the seller. Thus, a bargaining approach that incorporates the break-up clause could be another direction to explore. Third, we investigate the role of the termination fee paid by the seller (and eventually borne by the second buyer). In practice the reverse termination fee paid by the initial buyer in case of a buyer-induced breakup is also widely used. If we adopt the bifurcated rather than the one-side termination fee in the model, the go-shop provision can potentially raise more revenue because it removes the implicit "must-sell" constraint. In particular, the revenue superiority of static auction at the "no *ex post* information" extreme in section 4 should disappear. Fourth, go-shop clauses are often observed in buyer-initiated acquisitions. How deal formats are endogenously determined in a model with strategic deal initiation remains an open question.

Finally, our insights, although motivated by the selling mechanism in the M&A deals, are not limited to this particular market. The framework can be extended to a general analysis of break-up clauses in dynamic contract problems. Any market that involves contracts that features "termination fees" such as labor markets of artists or athletes, might share the same incentive structure.

References

- P. Aghion and P. Bolton. Contracts as a barrier to entry. *The American economic review*, pages 388–401, 1987.
- M. Akan, B. Ata, and J. D. Dana Jr. Revenue management by sequential screening. Journal of Economic Theory, 159:728–774, 2015.
- A. Antoniades, C. W. Calomiris, and D. M. Hitscherich. No free shop: Why target companies sometimes choose not to buy 'go-shop'options. *Journal of Economics and Business*, 88:36–64, 2016.
- M. Armstrong and J. Zhou. Search deterrence. The Review of Economic Studies, 83(1):26–57, 2015.
- T. W. Bates and M. L. Lemmon. Breaking up is hard to do? an analysis of termination fee provisions and merger outcomes. *Journal of Financial Economics*, 69(3):469–504, 2003.
- E. Berkovitch and N. Khanna. How target shareholders benefit from value-reducing defensive strategies in takeovers. *The Journal of Finance*, 45(1):137–156, 1990.
- E. Berkovitch, M. Bradley, and N. Khanna. Tender offer auctions, resistance strategies, and social welfare. Journal of Law, Economics, & Organization, 5(2):395–412, 1989.
- A. L. Boone and J. H. Mulherin. Do termination provisions truncate the takeover bidding process? The Review of Financial Studies, 20(2):461–489, 2007.
- J. Bulow and P. Klemperer. Why do sellers (usually) prefer auctions? American Economic Review, 99(4):1544–75, 2009.
- Y.-K. Che and T. R. Lewis. The role of lockups in takeover contests. The RAND Journal of Economics, 38(3):648–669, 2007.
- A. H. Choi. A rent extraction theory of right of first refusal. The Journal of Industrial Economics, 57(2):252–262, 2009.
- P. Courty and H. Li. Sequential Screening. The Review of Economic Studies, 67(4): 697–717, 2000.
- R. Deb and M. Said. Dynamic screening with limited commitment. Journal of Economic Theory, 159:891–928, 2015.

- V. Dequiedt. Efficient collusion in optimal auctions. Journal of Economic Theory, 136(1):302–323, 2007.
- T. Dimopoulos and S. Sacchetto. Preemptive bidding, target resistance, and takeover premiums. Journal of Financial Economics, 114(3):444–470, 2014.
- J. C. Ely, D. F. Garrett, and T. Hinnosaar. Overbooking. Journal of the European Economic Association, 15(6):1258–1301, 2017.
- P. Eső and B. Szentes. Optimal Information Disclosure in Auctions and the Handicap Auction. The Review of Economic Studies, 74(3): 705–731, 2007.
- P. Eso and B. Szentes. Dynamic contracting: An irrelevance theorem. *Theoretical Economics*, 12 (1):109–139, 2017.
- M. J. Fishman. A theory of preemptive takeover bidding. *The Rand Journal of Economics*, pages 88–101, 1988.
- D. F. Garrett. Intertemporal price discrimination: Dynamic arrivals and changing values. American Economic Review, 106(11):3275–99, 2016.
- D. F. Garrett. Dynamic mechanism design: dynamic arrivals and changing values. Games and Economic Behavior, 104:595–612, 2017.
- S. Gogineni and J. Puthenpurackal. The impact of go-shop provisions in merger agreements. Financial Management, 46(1):275–315, 2017.
- A. S. Gorbenko and A. Malenko. Strategic and financial bidders in takeover auctions. *The Journal of Finance*, 69(6):2513–2555, 2014.
- A. S. Gorbenko and A. Malenko. A theory of initiation of takeover contests. Available at SSRN 3014110, 2018.
- S. J. Grossman and O. D. Hart. Takeover bids, the free-rider problem, and the theory of the corporation. *The Bell Journal of Economics*, pages 42–64, 1980.
- R. L. Houtman and M. A. Morton. Go-shops: Market check magic or mirage. Potter Anderson & Corroon LLP Memorandum to Clients, pages 1–10, 2007.

- X. Hua. Strategic ex ante contracts: rent extraction and opportunity costs. The RAND Journal of Economics, 38(3):786–803, 2007.
- X. Hua. The right of first offer. International Journal of Industrial Organization, 30(4):389–397, 2012.
- J. Q. Jeon and C. Lee. Effective post-signing market check or window dressing? the role of go-shop provisions in m&a transactions. *Journal of Business Finance & Accounting*, 41(1-2):210–241, 2014.
- M. Kahan and M. Klausner. Lockups and the market for corporate control. Stan. L. Rev., 48:1539, 1995.
- M. H. Kotowski. On asymmetric reserve prices. Theoretical Economics, 13(1):205–237, 2018.
- D. Krähmer and R. Strausz. Optimal sales contracts with withdrawal rights. The Review of Economic Studies, 82(2):762–790, 2015.
- C. Li and X. Zhao. Job offer design with an endogenous acceptance deadline. Working Paper, 2019.
- J. Lu and Z. Wang. Optimal selling mechanisms with buyer price search. Working Paper, 2019.
- J. Lu and L. Ye. Orchestrating information acquisition. Working Paper, 2019.
- R. P. McAfee and J. McMillan. Auctions and bidding. *Journal of economic literature*, 25(2): 699–738, 1987.
- R. B. Myerson. Optimal auction design. Mathematics of operations research, 6(1):58-73, 1981.
- V. Nocke, M. Peitz, and F. Rosar. Advance-purchase discounts as a price discrimination device. Journal of Economic Theory, 146(1):141–162, 2011.
- M. S. Officer. Termination fees in mergers and acquisitions. Journal of Financial economics, 69(3): 431–467, 2003.
- A. Pavan, I. Segal, and J. Toikka. Dynamic mechanism design: A myersonian approach. *Econo-metrica*, 82(2):601–653, 2014.

- J. W. Roberts and A. Sweeting. When should sellers use auctions? *American Economic Review*, 103(5):1830–61, 2013.
- C. M. Sautter. Shopping during extended store hours: From no shops to go-shops-the development, effectiveness, and implications of go-shop provisions in change of control transactions. *Brook. L. Rev.*, 73:525, 2007.
- G. Subramanian. Go-shops vs. no-shops in private equity deals: Evidence and implications. Bus. Law., 63:729, 2007.
- G. Subramanian and A. Zhao. Go-shops revisited. Available at SSRN 3328202, 2019.
- C. Wang. Termination of dynamic contracts in an equilibrium labor market model. Journal of Economic Theory, 146(1):74–110, 2011.
- Z. Wang. Designing m&a selling mechanisms: Go-shop negotiations. Available at SSRN 3287321, 2018.

A Proofs

Proposition 3.1

Proof. (\Rightarrow) <u>Monotonicity</u>: Let $\tau > \tau'$. Incentive compatibility implies:

$$\begin{split} B(\tau,t(\tau)) + f(\tau) &\geq B(\tau,t(\tau')) + f(\tau') \\ B(\tau',t(\tau')) + f(\tau') &\geq B(\tau',t(\tau)) + f(\tau) \end{split}$$

Adding these two inequalities we have

$$B(\tau, t(\tau)) - B(\tau', t(\tau)) \ge B(\tau, t(\tau')) - B(\tau', t(\tau')) \iff \int_{t(\tau)}^{\bar{v}} H(v) \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv + H(t(\tau)) \int_{\underline{v}}^{t(\tau)} \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv \ge \int_{t(\tau')}^{\bar{v}} H(v) \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv + H(t(\tau')) \int_{\underline{v}}^{t(\tau')} \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv$$

Suppose for some pair $\tau > \tau'$ we have $t(\tau) < t(\tau'),$ we obtain:

$$\begin{split} \int_{t(\tau)}^{t(\tau')} H(v) \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv &\geq H(t(\tau')) \int_{\underline{v}}^{t(\tau')} \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv \\ &\qquad H(t(\tau)) \int_{\underline{v}}^{t(\tau)} \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv \\ &\geq H(t(\tau')) \int_{t(\tau)}^{t(\tau')} \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv \\ &= \int_{t(\tau)}^{t(\tau')} H(t(\tau')) \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv \\ &> \int_{t(\tau)}^{t(\tau')} H(v) \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv \end{split}$$

which leads to a contradiction. The second and third inequality both follow from $t(\tau) < t(\tau')$ and hence $H(t(\tau)) < H(t(\tau'))$.

Envelope theorem: We first prove the following lemma.

Lemma A.1. If the contract is incentive compatible, then U is increasing and Lipschitz continuous.

Proof. To prove monotonicity of $U(\tau)$, we consider any pair $\tau > \tau'$:

$$U(\tau) - U(\tau') \ge U(\tau, \tau') - U(\tau', \tau')$$

= $B(\tau, t(\tau')) - B(\tau', t(\tau'))$
= $\int_{t(\tau')}^{\bar{v}} H(v) [G_{\tau'}(v) - G_{\tau}(v)] dv + H(t(\tau')) \int_{\underline{v}}^{t(\tau')} [G_{\tau'}(v) - G_{\tau}(v)] dv$
 ≥ 0

The first inequality holds due to incentive compatibility. The last inequality follows from first order stochastic dominance.

To prove Lipschitz continuity, we observe that for any pair τ, τ' :

$$U(\tau) - U(\tau') \le U(\tau) - U(\tau', \tau) \le \sup_{\tilde{\tau}} \left\{ U(\tau, \tilde{\tau}) - U(\tau', \tilde{\tau}) \right\}$$
$$U(\tau') - U(\tau) \le U(\tau') - U(\tau, \tau') \le \sup_{\tilde{\tau}} \left\{ U(\tau', \tilde{\tau}) - U(\tau, \tilde{\tau}) \right\}$$

which implies

$$\begin{split} |U(\tau) - U(\tau')| &\leq \sup_{\tilde{\tau}} |U(\tau, \tilde{\tau}) - U(\tau', \tilde{\tau})| \\ &= \sup_{\tilde{\tau}} \Big| \int_{t(\tilde{\tau})}^{\bar{v}} H(v) \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv + H(t(\tilde{\tau})) \int_{\underline{v}}^{t(\tilde{\tau})} \left[G_{\tau'}(v) - G_{\tau}(v) \right] dv \Big| \\ &\leq \sup_{\tilde{\tau}} \int_{t(\tilde{\tau})}^{\bar{v}} H(v) \left| G_{\tau'}(v) - G_{\tau}(v) \right| dv + H(t(\tilde{\tau})) \int_{\underline{v}}^{t(\tilde{\tau})} \left| G_{\tau'}(v) - G_{\tau}(v) \right| dv \\ &\leq 2 \int_{\underline{v}}^{\bar{v}} \left| G_{\tau'}(v) - G_{\tau}(v) \right| dv \\ &\leq 2 \int_{\underline{v}}^{\bar{v}} \Big| \frac{\partial G_{\tau}(v)}{\partial \tau} \Big| |\tau' - \tau| dv \\ &\leq 2K |\bar{v} - \underline{v}| |\tau' - \tau| \end{split}$$

The last inequality follows from the bounded informativeness assumption. The second last inequality follows from the mean value theorem and that $G_{\tau}(v)$ is continuously differentiable in τ .

Since Lipschitz continuity implies absolute continuity, this lemma guarantees that U is differen-

tiable except in at most countably many points. By incentive compatibility, we consider any τ for which U is differentiable and let $\epsilon > 0$:

$$\lim_{\epsilon \to 0} \frac{U(\tau + \epsilon) - U(\tau)}{\epsilon} \ge \lim_{\epsilon \to 0} \frac{B(\tau + \epsilon, t(\tau)) + f(\tau) - B(\tau, t(\tau)) - f(\tau)}{\epsilon} = \frac{\partial B(\tau, t(\tau))}{\partial \tau}$$
$$\lim_{\epsilon \to 0} \frac{U(\tau) - U(\tau - \epsilon)}{\epsilon} \le \lim_{\epsilon \to 0} \frac{B(\tau, t(\tau)) + f(\tau) - B(\tau - \epsilon, t(\tau)) - f(\tau)}{\epsilon} = \frac{\partial B(\tau, t(\tau))}{\partial \tau}$$

Which implies that $U'(\tau) = \frac{\partial B(\tau, t(\tau))}{\partial \tau}$.

(\Leftarrow) To prove sufficiency we need to show that for each type τ there's no incentive to deviate to a contract for τ' :

$$U(\tau) \ge U(\tau, \tau') \Leftrightarrow$$
$$U(\tau) - U(\tau', \tau') \ge U(\tau, \tau') - U(\tau', \tau') \Leftrightarrow$$
$$U(\tau) - U(\tau', \tau') \ge B(\tau, \tau') - B(\tau', \tau') \Leftrightarrow$$
$$\int_{\tau'}^{\tau} \frac{\partial B(x, t(x))}{\partial x} dx \ge \int_{\tau'}^{\tau} \frac{\partial B(x, t(\tau'))}{\partial x} dx$$

The left hand side of the last inequality equals the left hand side of the second half due to the Envelope condition. Suppose $\tau > \tau'$, this inequality always holds since the two integrals have identical integration limits, $x \ge \tau'$ for all $x \in [\tau', \tau]$, $t(\tau)$ is increasing and that $\frac{\partial^2 B(\tau, t)}{\partial \tau \partial t}$ is always non-negative. The last argument is true because

$$\frac{\partial^2 B(\tau,t)}{\partial \tau \partial t} = h(t) \int_{\underline{v}}^t \left(-\frac{\partial G_{\tau}(v)}{\partial \tau} \right) dv \ge 0$$

where the inequality is guaranteed by first order stochastic dominance. We can prove the same result for $\tau < \tau'$ in the same way.

Proposition 3.2

Proof. The last proposition proved that $U(\tau)$ is increasing in τ , which implies that $U(\underline{\tau}) \ge 0 \Rightarrow U(\tau) \ge 0$.

Proposition 3.4

Proof. (3) is directly obtained from proposition 3.1. To prove (2) we rewrite the revenue term in equation 1:

$$\begin{split} R_{G} &= \int_{\underline{\tau}}^{\overline{\tau}} \left[t(\tau)C(\tau,t(\tau)) + D(\tau,t(\tau)) + B(\tau,t(\tau)) \right] dF(\tau) - \int_{\underline{\tau}}^{\overline{\tau}} \int_{\underline{\tau}}^{\underline{\tau}} \frac{\partial}{\partial x} B(x,t) dx dF(\tau) \\ &= \int_{\underline{\tau}}^{\overline{\tau}} \left[t(\tau)C(\tau,t(\tau)) + D(\tau,t(\tau)) + B(\tau,t(\tau)) \right] dF(\tau) - \int_{\underline{\tau}}^{\overline{\tau}} \int_{x}^{\overline{\tau}} dF(\tau) \frac{\partial}{\partial x} B(x,t) dx \\ &= \int_{\underline{\tau}}^{\overline{\tau}} \left[t(\tau)C(\tau,t(\tau)) + D(\tau,t(\tau)) + B(\tau,t(\tau)) \right] dF(\tau) - \int_{\underline{\tau}}^{\overline{\tau}} \left[1 - F(\tau) \right] \frac{\partial}{\partial \tau} B(\tau,t) d\tau \\ &= \int_{\underline{\tau}}^{\overline{\tau}} \left[t(\tau)C(\tau,t(\tau)) + D(\tau,t(\tau)) + B(\tau,t(\tau)) - \frac{1 - F(\tau)}{f(\tau)} \frac{\partial}{\partial \tau} B(\tau,t) \right] f(\tau) d\tau \end{split}$$

Let $R_G(\tau) = \left[t(\tau)C(\tau, t(\tau)) + D(\tau, t(\tau)) + B(\tau, t(\tau)) - \frac{1 - F(\tau)}{f(\tau)} \frac{\partial}{\partial \tau} B(\tau, t) \right]$ denote the integrand function. Notice that all terms in L are functions of $t(\tau)$, so by the Euler-Lagrange equation $\frac{\partial L}{\partial t} - \frac{d}{d\tau} \frac{\partial L}{\partial t'} = 0$ we get:

$$C(\tau,t) + t\frac{\partial C}{\partial t} + \frac{\partial D}{\partial t} + \frac{\partial B}{\partial t} - \frac{1 - F(\tau)}{f(\tau)}\frac{\partial}{\partial t}\frac{\partial B}{\partial \tau} = 0.$$

Plug in all the functional forms of $B(\tau, t(\tau)), C(\tau, t(\tau))$ and $D(\tau, t(\tau))$, we obtain:

$$\begin{split} 0 &= H(t) + G_{\tau}(t) - H(t)G_{\tau}(t) + t[h(t) + g_{\tau}(t) - h(t)G_{\tau}(t) - g_{\tau}(t)H(t)] \\ &- t(1 - G_{\tau}(t))h(t) - t(1 - H(t))g_{\tau}(t) - H(t)(1 - G_{\tau}(t)) - h(t)\int_{\underline{v}}^{t}G_{\tau}(v)dv - H(t)G_{\tau}(t) \\ &- \frac{1 - F(\tau)}{f(\tau)}\frac{\partial}{\partial t}\left[-\int_{t}^{\overline{v}}H(v)\frac{\partial G_{\tau}(v)}{\partial \tau}dv - H(t)\int_{\underline{v}}^{t}\frac{\partial G_{\tau}(v)}{\partial \tau}dv\right] \\ &= G_{\tau}(t) - H(t)G_{\tau}(t) - h(t)\int_{\underline{v}}^{t}G_{\tau}(v)dv + \frac{1 - F(\tau)}{f(\tau)}\left[h(t)\int_{\underline{v}}^{t}\frac{\partial G_{\tau}(v)}{\partial \tau}dv\right] \\ &= G_{\tau}(t)h(t)\left[\frac{1 - H(t)}{h(t)} - t + \frac{1}{G_{\tau}(t)}\int_{\underline{v}}^{t}\left(v + \frac{1 - F(\tau)}{f(\tau)}\frac{\partial G_{\tau}(v)/\partial \tau}{g_{\tau}(v)}\right)dG_{\tau}(v)\right]. \end{split}$$

where the third equality follows from integration by parts and implies equation (2). When $t = \underline{v}$, the first order derivative is 0 given the assumption that the informativeness measure $\partial G_{\tau}(v)/\partial \tau$ and all densities h, t_{τ} are bounded. When $t = \bar{v}$, the first order derivative is $h(\bar{v}) [E(\phi_1(v_1)) - \bar{v}] < 0$. Hence the first order derivative for each τ is either always non-positive and achieves the optimum at t = 0, or crosses 0 from above at least once, which guarantees the existence of the optimal solution.

Finally we show that $t(\tau)$ is increasing:

$$\frac{\partial^2 R_G(\tau)}{\partial t \partial \tau} = h(t) \left[-\psi_2(t) \frac{\partial G_\tau(t)}{\partial \tau} + \frac{\partial}{\partial \tau} \int_{\underline{v}}^t \left(v + \frac{1 - F(\tau)}{f(\tau)} \frac{\partial G_\tau(v)}{g_\tau(v)} \right) dG_\tau(v) \right]$$

Proposition 4.1

Proof. The optimal contract characterization follows directly from proposition 3.4. Here we prove the revenue ranking result by showing that $R_G^* \equiv R_G(t^*) \geq R_G(r^*) \geq R_S(r^*)$. The first inequality follows directly from the optimality of t^* . To prove the second inequality, it suffices to show:

$$\begin{aligned} rH(r)G(r) + B(r) &\geq 0 \\ \iff rH(r)G(r) + \int_{r}^{\bar{v}} H(v) \left[1 - G(v)\right] dv - H(r) \int_{\underline{v}}^{r} G(v) dv \geq 0 \\ \iff H(r) \left[rG(r) - \left(vG(v) \left| \frac{r}{\underline{v}} - \int_{\underline{v}}^{r} v dG(v) \right) \right] + \int_{r}^{\bar{v}} H(v) \left[1 - G(v)\right] dv \geq 0 \\ \iff H(r) \left[rG(r) - rG(r) + \int_{\underline{v}}^{r} v dG(v) \right] + \int_{r}^{\bar{v}} H(v) \left[1 - G(v)\right] dv \geq 0 \\ \iff H(r) \int_{\underline{v}}^{r} v dG(v) + \int_{r}^{\bar{v}} H(v) \left[1 - G(v)\right] dv \geq 0. \end{aligned}$$

Proposition 4.2

Proof. The conditions follows proposition 3.4 directly. When G = H, by regularity condition we obtain $t(v) = v \ \forall v$. In this case the price in the optimal go-shop negotiation is always t(v) = v for any bidder 1's value v, hence the allocation rule is the same as an ordinary auction. Furthermore

the IR constraints that the lowest type gets zero payoffs in both mechanisms imply that they are revenue equivalent. $\hfill \Box$

Proposition 4.3

Proof. Take first order derivation of revenue term R_G with respect to a, we get

$$\begin{aligned} \frac{\partial R_G(a)}{\partial a} &= \int_{\tau} \left\{ t(\tau) \frac{\partial C}{\partial a} + \frac{\partial D}{\partial a} + \frac{\partial B}{\partial a} - \frac{1 - F(\tau)}{f(\tau)} \frac{\partial}{\partial a} \frac{\partial B}{\partial \tau} \right\} dF(\tau) \\ &= \int_{\tau} \left\{ -H(t) \int_{\underline{v}}^t \frac{\partial G(v|\tau, a)}{\partial a} dv - \int_t^{\overline{v}} \frac{\partial G(v|\tau, a)}{\partial a} dv + H(t) \int_{\underline{v}}^t \frac{\partial^2 G(v|\tau, a)}{\partial \tau \partial a} dv + \int_t^{\overline{v}} H(v) \frac{\partial^2 G(v|\tau, a)}{\partial \tau \partial a} dv \right\} dF(\tau) \\ &= \int_{\tau} \left\{ H(t) \int_{\underline{v}}^t \frac{\partial^2 G(v|\tau, a)}{\partial \tau \partial a} dv + \int_t^{\overline{v}} H(v) \frac{\partial^2 G(v|\tau, a)}{\partial \tau \partial a} dv \right\} dF(\tau) \end{aligned}$$

It's straightforward that $\frac{\partial^2 G(v|\tau,a)}{\partial \tau \partial a} \leq 0 \ \forall \tau, a, v$ is sufficient for R_G to be decreasing in a.

B Binary Type Binary Value

Suppose $v_i = \{v_l, v_h\}$ for $i \in \{1, 2\}$ where $0 < v_l < v_h < \infty$. The conditional distributions of B1's final value are binary. Let α denote the probability that B1 has a high type. Let $\rho_h, \rho_l \in [\frac{1}{2}, 1]$ be the probability that the initial high (low) types has a high (low) value realization. B2 has a high value with probability β .

To break the tie, we make a modification to the auction form in the binary setting as follows. In the bidding stage, each buyer submits a bid and the early buyer is required to send an identity message $m \in \{v_l, v_h\}$ in case of a tie. The early buyer wins if and only if $b_1 + f(\tau) > b_2$ or $b_1 + f(\tau) = b_2$ and m = h. When B1 wins, he pays $\max\{b_2 - f, \underline{b}(\tau)\}$. When buyer 2 wins, he pays $b_1 + f$. The following lemma summarizes the bidding equilibrium.

Lemma B.1. In equilibrium B2 bids $b_2 = v_2 - f(\tau)$. B1 bids $b_1 = \max\{\underline{b}(\tau), v_1 - f(\tau)\}$ and reports identity truthfully whenever required.

Proof. The only nontrivial case of a tie is when $\underline{b}(\tau) + f(\tau) = v_h$. In such a case, buyer 1 always bids $b_1 = \underline{b}(\tau)$ and knows that $b_2 = v_2 = v_h$. If $v_1 = v_l$, buyer 1 loses the item and gets $f(\tau)$

by reporting identity truthfully and gets the item but incurs a negative net payoff $v_l - \underline{b}(\tau)$. If $v_1 = v_h$, buyer 1 gets $v_h - \underline{b}(\tau) = f(\tau)$ regardless of the message, and hence is willing to report truthfully.

The identity message is merely to break the potential tie in favor of the seller. Without such an augmentation the seller cannot distinguish between the state (v_l, v_h) and (v_h, v_h) , and the tie breaking rule matters.

In the contracting stage, the seller's problem turns out to be a linear programming:

$$\begin{split} \max_{\underline{b}_{\tau},f_{\tau}} R^{GS} &= \alpha [\rho_h \beta (v_h - f_H) + (1 - \rho_h \beta) \underline{b}_H] + (1 - \alpha) [(1 - \rho_l) \beta (v_h - f_L) + (1 - \beta + \rho_l \beta) \underline{b}_L] \\ s.t. \ [IC_L] : (1 - \beta) (\rho_l v_l + (1 - \rho_l) v_h - \underline{b}_L) + \beta f_L \geq (1 - \beta) (\rho_l v_l + (1 - \rho_l) v_h - \underline{b}_H) + \beta f_H \\ [IC_H] : (1 - \beta) ((1 - \rho_h) v_l + \rho_h v_h - \underline{b}_H) + \beta f_H \geq (1 - \beta) ((1 - \rho_h) v_l + \rho_h v_h - \underline{b}_L) + \beta f_L \\ [IR_L] : (1 - \beta) (\rho_l v_l + (1 - \rho_l) v_h - \underline{b}_L) + \beta f_L \geq 0 \\ [IR_H] : (1 - \beta) ((1 - \rho_h) v_l + \rho_h v_h - \underline{b}_H) + \beta f_H \geq 0 \\ [FE] : \underline{b}_L + f_L \in [v_l, v_h], \ \underline{b}_H + f_H \in [v_l, v_h], \ b_L, b_H, f_L, f_H \geq 0 \end{split}$$

Proposition B.1. The optimal go-shop mechanism in the binary environment is given by:

$$b_{H}^{*} = b_{L}^{*} = (1 - \rho_{l} + \rho_{l}\beta)v_{h} + (1 - \beta)\rho_{l}v_{l}$$
$$f_{H}^{*} = f_{L}^{*} = \rho_{l}(1 - \beta)(v_{h} - v_{l})$$
$$R^{G*} = (1 - \rho_{l} + \rho_{l}\beta)v_{h} + (1 - \beta)\rho_{l}v_{l}.$$

Proof. We refer to e_l and e_h as the expected value for low and high initial type. IC_L and IC_H together imply:

$$(1-\beta)\underline{b}_H - \beta f_H = (1-\beta)\underline{b}_L - \beta f_L.$$

Given this inequality IR_H is further implied by IR_L since $\rho \geq \frac{1}{2}$ and $v_h > v_l$. In addition, IC_L must bind at the optimum since otherwise the seller can reduce termination fee f_L and f_H by the

same amount to increase revenue while respecting all relevant constraints. So we have the equality:

$$(1-\beta)\underline{b}_H - \beta f_H = (1-\beta)\underline{b}_L - \beta f_L = (1-\beta)e_l.$$

Substitute this equality into the objective function and the feasibility constraints, we can rewrite the seller's problem as

$$\max_{\underline{b}_{H},\underline{b}_{L}} R^{G} = \alpha (1-\rho_{h}) \underline{b}_{H} + (1-\alpha)\rho_{l} \underline{b}_{L} + [\alpha\rho_{h} + (1-\alpha)(1-\rho_{l})]((1-\beta)e_{l} + \beta v_{h})$$

s.t. $\underline{b}_{L} + \frac{1-\beta}{\beta} (\underline{b}_{L} - e_{l}) \in [v_{l}, v_{h}], \ \underline{b}_{H} + \frac{1-\beta}{\beta} (\underline{b}_{H} - e_{l}) \in [v_{l}, v_{h}], \ b_{L}, b_{H} \ge e_{l}.$

Simplifying the constraint we can easily get the optimizing floor prices $\underline{b}_H = \underline{b}_L = \beta v_h + (1 - \beta)e_l$ and termination fees $f_H = f_L = (1 - \beta)(v_h - e_l)$. The value of the objective function is $R^G = \beta v_h + (1 - \beta)e_l$.

The optimal (symmetric) English auction in the current example takes one of the following two forms: the seller sets reserve price at either v_l or v_h . The maximal revenue collected in such an auction is $R^S = \max\{\beta\gamma v_h + (1-\beta\gamma)v_l, (\gamma+\beta-\gamma\beta)v_h\}$ where $\gamma = \alpha\rho_h + (1-\alpha)(1-\rho_l)$ denotes the prior probability that buyer 1 has a high value realization. The following proposition compares the optimal go-shop negotiation and the optimal English auction. The condition that is sufficient and necessary for the revenue superiority of go-shop negotiation is the same. One distinction is that, in contrast to the continuous value case, in the binary setting the symmetric English auction does not always implement the optimal static direct mechanism, even if the value distribution is identical.¹⁸

Proposition B.2. The optimal symmetric English auction sets reserve price at either $r = v_l$ or $r = v_h$ and raises revenue $R^{SE} = \max\{\beta\gamma v_h + (1 - \beta\gamma)v_l, (\gamma + \beta - \gamma\beta)v_h\};$

The optimal go-shop negotiation collects higher revenue than optimal English auction if and only if $\frac{v_h - v_l}{v_l} \leq \frac{1 - \gamma}{\gamma - (1 - \rho_l)}$;

The optimal static mechanism sets reserve price at either $r = v_h$, or $(r_1 = v_h, r_2 = v_l)$ and bidder 1 wins if and only if $b_1 = v_h$, or $(r_1 = v_l, r_2 = v_h)$ and bidder 1 wins if and only if $b_2 = v_l$, and raises revenue $R^E = \max\{\beta v_h + (1 - \beta)v_l, \gamma v_h + (1 - \gamma)v_l, (\gamma + \beta - \gamma \beta)v_h\};$

 $^{^{18} \}rm{See}$ Dequiedt (2007) for a derivation of the optimal static mechanism, and Kotowski (2018) for an implementation with two distinct reserve prices.

The optimal go-shop negotiation collects higher revenue than optimal English auction if and only if $\max\{\frac{v_h-v_l}{v_l}, \frac{1-\beta}{\beta}\} \leq \frac{1-\gamma}{\gamma-(1-\rho_l)}.$

Proof. The optimal go-shop negotiation raises higher revenue than optimal symmetric English auction if and only if

$$\beta v_h + (1 - \beta)e_l \ge \beta \gamma v_h + (1 - \beta \gamma)v_l \iff \rho_l \le \frac{1 - \gamma \beta}{1 - \beta} < 1$$

which always holds and

$$\beta v_h + (1 - \beta)e_l \ge (\gamma + \beta - \gamma\beta)v_h \Longleftrightarrow \frac{v_h}{v_l} \le \frac{\rho_l}{\alpha(\rho_h + \rho_l - 1)} \Longleftrightarrow \frac{v_h - v_l}{v_l} \le \frac{1 - \gamma}{\gamma - (1 - \rho_l)}$$

The optimal go-shop negotiation raises higher revenue than optimal English auction if and only if

$$\beta v_h + (1 - \beta)e_l \ge \beta v_h + (1 - \beta)v_l \iff \rho_l \le 1$$

which always holds and

$$\beta v_h + (1-\beta)e_l \ge \gamma v_h + (1-\gamma)v_l \iff \frac{1}{\beta} \le \frac{\rho_l}{\alpha(\rho_h + \rho_l - 1)} \iff \frac{1-\beta}{\beta} \le \frac{1-\gamma}{\gamma - (1-\rho_l)}$$
$$\beta v_h + (1-\beta)e_l \ge (\gamma + \beta - \gamma\beta)v_h \iff \frac{v_h}{v_l} \le \frac{\rho_l}{\alpha(\rho_h + \rho_l - 1)} \iff \frac{v_h - v_l}{v_l} \le \frac{1-\gamma}{\gamma - (1-\rho_l)}.$$

To see the intuition behind proposition B.2, we consider a symmetric example where $\rho_l = \rho_h = \rho$ and $\alpha = \beta = \gamma = \frac{1}{2}$. In the static mechanism, the virtual valuation of a v_h -bidder is v_h , while the virtual valuation of a v_l -bidder is $\left(v_l - \frac{\gamma}{1-\gamma}(v_h - v_l)\right)$. This is simply the well-known "no distortion at the top" result in standard contract theory. The downward distortion for the low value is to account for the information rent that the seller has to provide. The seller sells the good at all states whenever $v_l - \frac{\gamma}{1-\gamma}(v_h - v_l) \ge 0$, or $\frac{v_h - v_l}{v_l} \le \frac{1-\gamma}{\gamma}$, and collects revenue $\gamma v_h + (1-\gamma)v_l$. This outcome can be implemented by an English auction with asymmetric reserve price $(r_1 = v_h, r_2 = v_l)$ and asymmetric tie-breaking rule where bidder 1 wins if and only if $b_1 = v_h$. When the opposite holds, $\frac{v_h - v_l}{v_l} > \frac{1-\gamma}{\gamma}$, the seller only sells the good to a v_h -bidder at reserve price v_h and collects revenue $(1 - (1 - \gamma)^2)v_h$. The cutoff condition can be interpreted as "whether the low value is high enough so that the seller is willing to compromise the information rent to the v_h -bidder in exchange for the transaction at state (v_l, v_l) ".

Corollary B.1. When ρ approaches 1, the optimal go-shop contract approaches full extraction from bidder 2. When ρ approaches $\frac{1}{2}$, the optimal go-shop contract approaches the first-best outcome.

Corollary B.2. The optimal go-shop negotiation always achieves fully efficiency, extracts all surplus from the late buyer and the low type early buyer, and admits positive surplus to the high type early buyer.

$$TW = TW^{FB}, \Pi_{1h} = (1 - \beta)(\rho_h + \rho_l - 1)(v_h - v_l), \Pi_{1l} = 0, \Pi_2 = 0$$

The optimal static mechanism achieves full efficiency only if v_l has positive virtual value, and admits positive surplus to the weaker buyer only when full efficiency is attained.

$$TW = \begin{cases} TW^{FB} & \text{if } \frac{v_h - v_l}{v_l} \le \max\{\frac{1 - \beta}{\beta}, \frac{1 - \gamma}{\gamma}\}\\ (\beta + \gamma - \beta\gamma)v_h & \text{otherwise} \end{cases},$$
$$\Pi_1 = \begin{cases} \gamma(1 - \beta)(v_h - v_l) & \text{if } \frac{1 - \beta}{\beta} \le \min\{\frac{1 - \gamma}{\gamma}, \frac{v_h - v_l}{v_l}\}\\ 0 & \text{otherwise} \end{cases}$$
$$\Pi_2 = \begin{cases} \beta(1 - \gamma)(v_h - v_l) & \text{if } \frac{1 - \gamma}{\gamma} \le \min\{\frac{1 - \beta}{\beta}, \frac{v_h - v_l}{v_l}\}\\ 0 & \text{otherwise} \end{cases}$$

It is easy to see that we can focus on the cases where both the "floor price" \underline{b}_{τ} and the "deterrence price" t_{τ} are within $[v_l, v_h]$. Hence, the pricing equilibrium is described as follows: both bidders shade their bids by f; B1 wins whenever $v_2 = v_l$, B2 wins when $v_2 = v_h$ and $v_1 = v_l$, and each bidder wins with equal probability when $v_1 = v_2 = v_h$.

We can illustrate the main trade-off for the revenue maximizing seller. Recall that in an optimal static auction the seller faces the trade-off between efficiency and rent extraction: introducing an effective reserve price distorts total allocative efficiency due to no-sale in the "low-low" state, but increases rent extraction from the winning bidder when the reserve price binds. The optimal reserve

price is set to balance such marginal benefit and cost. In the binary setting, the optimal reserve price is a corner solution $(r = v_h)$: any reserve price $r \in (v_l, v_h]$ induces the same loss of sale in the "low-low" state, while higher r increases the final price in other states. In particular, this extreme reserve price extracts all bidder surplus.

In the go-shop mechanism, however, the item is always sold.¹⁹ The floor price sets the effective price in all events but $(v_1 = v_2 = v_h)$. However, the seller cannot increase sale in the low-low state without incurring any cost: the price is reduced to an interior level <u>b</u>, and furthermore the seller pays the high type B1 whenever he loses. This result seems surprising: since the reserve price $r = v_h$ is already the best way to balance the efficiency vs. rent extraction trade-off, how can the seller benefit from higher efficiency at the cost of positive bidder surplus? The reason becomes clear if we look at the consumer surplus of B1 in the optimal contract: the low type earns expected surplus of 0 while the high type gets positive expected surplus; the low value B1, however, incurs a loss $(v_l - \underline{b}(\tau) < 0)$ in the bidding stage regardless of the initial type. In other words, the go-shop negotiation only imposes interim participation constraints while the static optimal auction is restricted by *ex post* participation constraints.²⁰

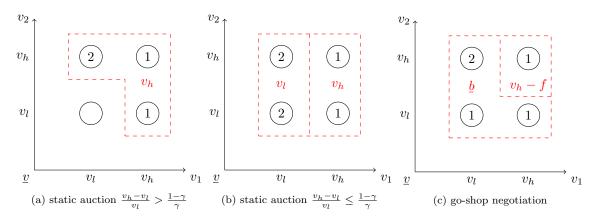


Figure 6: Equilibrium allocation in the bidding stage

¹⁹In this binary example it also attains full efficiency. It is not, however, in general true since it might still distort allocation between the two buyers: given the priority the first buyer might win while holding a lower ex-post value than the second buyer.

 $^{^{20}}$ The same point is demonstrated in detail in Eső and Szentes (2007) and Eso and Szentes (2017). In the fully optimal dynamic model, B1's *ex post* informational advantage does not bring him additional information rent since the contract is signed before the final value is observed. The only bidder rent comes from the *ex ante* private information.